Student's Solutions Manual

to accompany

TRIGONOMETRY

Third Edition

Wesner • Mahler

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Student Solutions Manual to Accompany Trigonometry with Applications

Introduction to the Student

When you study mathematics you are doing something which will help you for the rest of your life. Although most students do not realize it mathematics is used in almost every discipline which a student is likely to enter, from nuclear physics to music. In fact, in the age of electronic calculating devices mathematics is more important than ever.

Assumptions upon which the text was created

In writing this text we have assumed that you have completed an intermediate algebra course, and therefore has been introduced to solving equations, factoring, radicals and graphing linear equations. This means that the language of algebra should not be new to you. The following problems and their solutions should not seem entirely foreign to you, even if you have forgotten some of the details.

1. Solve the equation $5x - 3 = 2(x + 7)$

   Solution:

   $5x - 3 = 2x + 14$

   $5x - 2x - 3 = 14$ — Subtract $2x$ from each member

   $3x - 3 = 14$ — $5x - 2x = 3x$

   $3x = 17$ — Add 3 to both members

   $x = \frac{17}{3}$ — Divide each member by 3

2. Factor $3x^2 - 2x - 4$

   Solution:

   $3x^2 - 9x - 4$

   $(3x \quad)(x \quad)$ — $3x \cdot x = 3x^2$

   $(3x \pm 4)(3x \pm 1)$

   $(3x \pm 2)(3x \pm 2)$ — Several ways to get -4 in the third term

   $(3x - 4)(3x + 1)$ — The choice which gives $-9x$ for the middle term

3. Simplify $\sqrt{8x^5}$

   Solution:

   $\sqrt{8x^5} = \sqrt{2^2 \cdot 2 \cdot x^4 \cdot x}$ — Factor

   $= 2x^2\sqrt{2x}$ — $\sqrt{2^2} = 2$; $\sqrt{x^4} = x^2$

<u>Throughout the text we will remind you about the details of this material as needed.</u>

It is also assumed that you own a scientific calculator or graphing calculator, and are familiar with the basic keys for arithmetic computation. Keystrokes for a typical scientific calculator are presented in the text where they go beyond the basic arithmetic operations.

Graphing Calculators/Computers

You may own a graphing calculator instead of a scientific calculator. The text specifically indicates how to use these devices. Examples are presented based on the TEXAS INSTRUMENTS TI–81 graphing calculator. The introductory section *Computer Aided Mathematics* introduces the basic principles involved in using a graphing calculator, using the TI–81 as an example.

Tips for Success

- **Work** Success in a mathematics course requires a lot of work! You must work assigned problems all the time. Much of what you will learn is complicated and needs constant practice.
- **Regular study time** Try to set aside some time every day in which you will do some mathematics. If you have already finished all assigned problems then go back to previous sections of the text and do some review problems. Certain sections will have seemed harder than others. Go back to these harder sections! With enough work you will understand everything. Remember what Thomas Edison said. "Genius is 10% inspiration and 90% perspiration."
- **You can do it!** It is true that some people have a "knack" for mathematics, just as some do for tennis, basketball, music, drawing, or English composition. However, this does not mean that you cannot do these things if you don't have the knack – it just means that you have to work at it. No one gets good at anything without working at it. If they don't seem to work at something it's just because they love it and don't consider what they do to be work.
- **How to study** Read the text seated at a desk. Have a pencil and paper at hand. When you come to an example, copy the steps onto your paper as you read. This will help you understand better for several reasons.

  – Mathematics looks different when it is written by hand than when it is printed. You should get used to what it looks like when you write it!

  –Some ways are better than others to organize mathematics problems. We try to show you a good way to do this in the text, and you want to get used to it.

  –It has been shown that one learns best when more than one sense is used. Writing uses motor skills as well as sight.
- **Do the homework** It has been said that mathematics is not a spectator sport. You could watch someone play the piano for years, but this would not help you learn to play the piano. You must do it yourself.
- **Mathematics Ability is not in the Genes** – *Anyone can do mathematics.* Many people tend to feel that difficulties in learning mathematics indicate lack of ability, and that mathematics should therefore be avoided by an individual encountering problems. The answer to difficulties is to work harder, not to give up. Mathematics is very important throughout most professions, and it just won't go away!

Features of the text to help you learn

Problem Sets

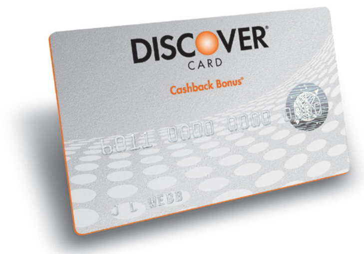
- The drill portion of the problem sets are similar to the examples of that section.
- The *answers to all problems* appear at the end of the text, except for even–numbered exercises. The *complete solutions to selected problems*, indicated by boxing the problem number, also appear at the end of the text. If you have trouble with a particular problem you should be able to find the solution to a similar problem either in the examples in the text, the selected (box numbered) problems, or in this manual.
- The exercises progress from straightforward application of the material covered in the exposition to problem solving via more difficult application problems and then to problems which require some ingenuity and creativity. Those problems requiring exceptional ingenuity or amount of work are marked with the symbol ▢. If you try the complete range of problems you will have a good introduction to the way in which this material is used in a variety of disciplines.

Chapter Summary, Review, Test at the end of each chapter

- Chapter Summary – This reviews the highlights and key points of the chapter. Read this over after the chapter is completed. Think about each item. If something seems unfamiliar go and find it in the text and review it.
- Chapter Review – This presents review problems from the chapter, keyed to sections. You should work this after the chapter is completed.
- Chapter Test – This is designed to help you practice the material as it might appear on a test, out of the context of each section. The chapter test may well be longer than what would be given in class. The test provides material out of the context provided by knowing what section it is from, and by being surrounded by similar problems. In the homework sets there are inevitably many clues to the method of solution, including nearby problems and temporal and physical proximity to explanations. The chapter test is an aid to make that last link in learning – recognition of problem type, with attending method of solution.

Applications — Applications problems are put in the text to show you where mathematics is used in the various disciplines. Do not be afraid of them. You will see that you don't have to know, for example, electronics to do those applications which apply to electronics. The problems always make clear what mathematical operations and principles are involved.

Work hard and you will enjoy your education
and succeed at it.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Chapter 1

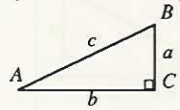
Exercise 1–1

1. $13° \, 25' = \left(13 + \frac{25}{60}\right)° \approx 13.417°$; acute

5. $25° \, 33' \, 19'' =$
$\left(25 + \frac{33}{60} + \frac{19}{3600}\right)° \approx 25.555°$; acute

9. $33° \, 5' \, 55'' =$
$\left(33 + \frac{5}{60} + \frac{55}{3600}\right)° \approx 33.099°$; acute

13.



17. $90° - 18° \, 12' = 89° \, 60' - 18° \, 12' =$
$71° \, 48'$

21. $.3^2 + .4^2 = .5^2$ so it is a right triangle. Hypotenuse is 0.5

25. $0.32^2 + 2.55^2 = 2.57^2$, so this is a right triangle with hypotenuse 2.57.

29.
$c^2 = a^2 + b^2$
$10^2 = a^2 + 8^2$
$100 = a^2 + 64$
$36 = a^2$
$a = \sqrt{36} = 6$

33.
$c^2 = a^2 + b^2$
$c^2 = (\sqrt{5})^2 + 3^2$
$c^2 = 5 + 9$
$c^2 = 14$
$c = \sqrt{14} \approx 3.7 \approx 4$

37.
$c^2 = a^2 + b^2$
$c^2 = (\sqrt{7})^2 + 3^2$
$c^2 = 7 + 9$
$c^2 = 16$
$c = 4$

41.
$c^2 = a^2 + b^2$
$c^2 = (3\sqrt{2})^2 + (4\sqrt{5})^2$
$c^2 = 9 \cdot 2 + 16 \cdot 5$
$c^2 = 98$
$c = \sqrt{98}$
$c = 7\sqrt{2} \approx 9.9$

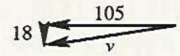
45.
$c^2 = a^2 + b^2$
$c^2 = 1^2 + 1^2$
$c^2 = 2$
$c = \sqrt{2} \approx 1.4 \approx 1$

49.
$213^2 = 193^2 + w^2$
$8120 = w^2$
90.1 feet $\approx w$

53.
$Z^2 = R^2 + X_L^2$
$4340^2 = R^2 + 2150^2$
$14213100 = R^2$
3770 ohms $\approx R$

57. $d^2 = 15^2 + 10.5^2$, so $d \approx 18.3$ cm, and $\frac{18.3 \text{ cm}}{0.8 \text{ cm/min}} = 23$ minutes to make the cut.

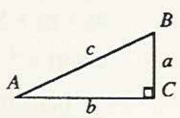
61. $v^2 = 18^2 + 105^2 = 11349$
$v \approx 107$ knots



65.
$B^2 = a^2 + b^2$
$C^2 = B^2 + c^2 = a^2 + b^2 + c^2$
$D^2 = C^2 + d^2$
$\quad = a^2 + b^2 + c^2 + d^2$
$E^2 = D^2 + e^2$
$\quad = a^2 + b^2 + c^2 + d^2 + e^2$
$E^2 = 20^2 + 5^2 + 12^2 + 8^2 + 10^2$
$E^2 = 733$
$E \approx 27$

Exercise 1-2

1.
$\sin A = \frac{a}{c}$



$\csc A = \frac{c}{a}$
$\cos A = \frac{b}{c}$ $\quad \sec A = \frac{c}{b}$
$\tan A = \frac{a}{b}$ $\quad \cot A = \frac{b}{a}$
$\sin B = \frac{b}{c}$ $\quad \csc B = \frac{c}{b}$
$\cos B = \frac{a}{c}$ $\quad \sec B = \frac{c}{a}$
$\tan B = \frac{b}{a}$ $\quad \cot B = \frac{a}{b}$

5.
$c^2 = 1^2 + 3^2$
$c = \sqrt{10}$
$\sin B = \frac{b}{c} = \frac{3}{\sqrt{10}} = \frac{3}{10}\sqrt{10}$
$\csc B = \frac{\sqrt{10}}{3}$
$\cos B = \frac{a}{c} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$
$\sec B = \sqrt{10}$
$\tan B = \frac{b}{a} = \frac{3}{1} = 3$ $\quad \cot B = \frac{1}{3}$

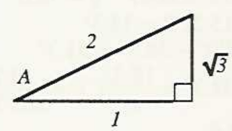
9.
$2^2 + b^2 = (\sqrt{5})^2$
$b^2 = 1$
$b = 1$
$\sin B = \frac{b}{c} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$
$\csc B = \sqrt{5}$
$\cos B = \frac{a}{c} = \frac{2}{\sqrt{5}} = \frac{2}{5}\sqrt{5}$
$\sec B = \frac{\sqrt{5}}{2}$
$\tan B = \frac{b}{a} = \frac{1}{2}$ $\quad \cot B = \frac{2}{1} = 2$

13.
$6^2 + b^2 = 10^2$
$b = 64$
$b = 8$
$\sin A = \frac{a}{c} = \frac{6}{10} = \frac{3}{5}$ $\quad \csc A = \frac{5}{3}$
$\cos A = \frac{b}{c} = \frac{8}{10} = \frac{4}{5}$ $\quad \sec A = \frac{5}{4}$
$\tan A = \frac{a}{b} = \frac{6}{8} = \frac{3}{4}$ $\quad \cot A = \frac{4}{3}$

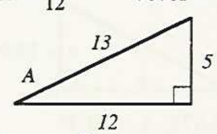
17.
$x^2 + b^2 = z^2$
$b^2 = z^2 - x^2$
$b = \sqrt{z^2 - x^2}$
$\sin B = \frac{b}{c} = \frac{\sqrt{z^2 - x^2}}{z}$
$\csc B = \frac{z}{\sqrt{z^2 - x^2}}$
$\cos B = \frac{a}{c} = \frac{x}{z}$
$\sec B = \frac{c}{a} = \frac{z}{x}$
$\tan B = \frac{b}{a} = \frac{\sqrt{z^2 - x^2}}{x}$
$\cot B = \frac{x}{\sqrt{z^2 - x^2}}$

21.
$9^2 + 5^2 = c^2$
$106 = c^2$
$\sqrt{106} = c$
$\sin B = \frac{b}{c} = \frac{5}{\sqrt{106}} \cdot \frac{\sqrt{106}}{\sqrt{106}}$
$= \frac{5}{106}\sqrt{106}$ $\quad \csc B = \frac{\sqrt{106}}{5}$
$\cos B = \frac{a}{c} = \frac{9}{\sqrt{106}} = \frac{9}{106}\sqrt{106}$
$\sec B = \frac{\sqrt{106}}{9}$
$\tan B = \frac{b}{a} = \frac{5}{9}$ $\quad \cot B = \frac{9}{5}$

25.
$\cos A = 0.5 = \frac{1}{2}$ $\quad \sec A = 2$
$\sin A = \frac{\sqrt{3}}{2}$ $\quad \csc A = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$
$\tan A = \sqrt{3}$ $\quad \cot A = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

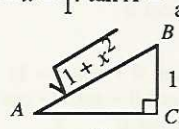


29.
$\sin A = \frac{5}{13}$ $\quad \csc A = \frac{13}{5}$
$\cos A = \frac{12}{13}$ $\quad \sec A = \frac{13}{12}$
$\tan A = \frac{5}{12}$ $\quad \cot A = \frac{12}{5}$



33.
$\sin^2 A + \cos^2 A$
$(\frac{a}{c})^2 + (\frac{b}{c})^2$
$\frac{a^2}{c^2} + \frac{b^2}{c^2}$
$\frac{a^2 + b^2}{c^2}$
$\frac{c^2}{c^2}$
1

37. $\tan B = x = \frac{x}{1}$. $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{1}{x}$



Exercise 1-3

1. $\sin 31.28° \approx 0.5192$
5. $\cot 28.87° \approx 1.8137$

   28.87 $\boxed{\tan}$ $\boxed{1/x}$

   $\boxed{\text{TI-81}}$ $\boxed{(}$ $\boxed{\text{TAN}}$ 28.87 $\boxed{)}$

   $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$
9. $\sec 66.47° \approx 2.5048$
13. $\sin 78° 33' \approx 0.9801$

    78 $\boxed{+}$ 33 $\boxed{\div}$ 60 $\boxed{=}$ $\boxed{\sin}$

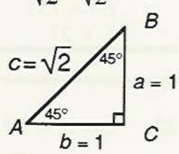
    $\boxed{\text{TI-81}}$ $\boxed{\text{SIN}}$ $\boxed{(}$ 78 $\boxed{+}$ 33 $\boxed{\div}$ 60

    $\boxed{)}$ $\boxed{\text{ENTER}}$
17. $\tan 35° 8' \approx 0.7037$
21. $\sin 48° 8' \approx 0.7447$
25. $R = \dfrac{611.1}{2 \sin 18° 20'} \approx 971.40$ sq. meters
29. $P = 42 \cdot 25 \cdot \cos 45° \approx 742.5$ watts
33. Using the figure, we find $c = \sqrt{2}$. Then

    $\sin 45° = \sin A = \dfrac{a}{c} = \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

    $\cos 45° = \cos A$

    $= \dfrac{b}{c} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

    $\tan 45° = \tan A =$

    $\dfrac{a}{b} = \dfrac{1}{1} = 1$
37. $\tan \theta = 1.8807 \quad \theta \approx 62.0°$
41. $\sin \theta = \dfrac{35.9}{68.3} \quad \theta \approx 31.7°$
45. $\sec \theta = 4.8097 \quad \theta \approx 78.0°$
49. $a = 15.2, B = 38.3°$

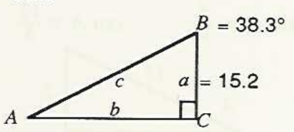
    $A = 90° - 38.3° = 51.7°$

    $\cos 38.3° = \dfrac{15.2}{c}; \; c = \dfrac{15.2}{\cos 38.3°}$

    ≈ 19.4

    $\tan 38.3° = \dfrac{b}{15.2}; b = 15.2 \tan 38.3°$

    ≈ 12.0

53. $b = 0.672, A = 29.4°$

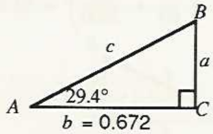
    $B = 90° - 29.4° = 60.6°$

    $\tan 29.4° = \dfrac{a}{0.672}; a = 0.672 \tan 29.4°$

    ≈ 0.379

    $\cos 29.4° = \dfrac{0.672}{c}; c = \dfrac{0.672}{\cos 29.4°}$

    ≈ 0.771

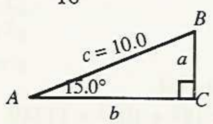


57. $c = 10.0, A = 15.0°$

    $B = 90° - 15.0° = 75.0°$

    $\cos 15° = \dfrac{b}{10}; b = 10 \cos 15° \approx 9.7$

    $\sin 15° = \dfrac{a}{10}; a = 10 \sin 15° \approx 2.6$

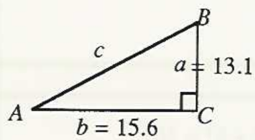
    

61. $a = 13.1 \quad b = 15.6$

    $c^2 = \sqrt{13.1^2 + 15.6^2}; c \approx 20.4$

    $\tan A = \dfrac{13.1}{15.6}; A \approx 40.0°$

    $\tan B = \dfrac{15.6}{13.1}; B \approx 50.0°$

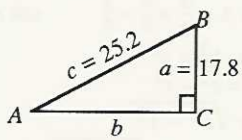
65. $a = 17.8 \quad c = 25.2$

    $17.8^2 + b^2 = 25.2^2$

    $b = \sqrt{25.2^2 - 17.8^2} \approx 17.8$

    $\sin A = \dfrac{17.8}{25.2}; A \approx 44.9°$

    $\cos B = \dfrac{17.8}{25.2}; B \approx 45.1°$

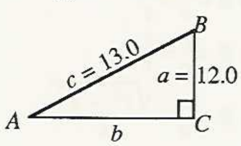
    

69. $a = 12.0 \quad c = 13.0$

    $b = \sqrt{13^2 - 12^2} \approx 5.0$

    $\sin A = \dfrac{12}{13}; A \approx 67.4°$

    $\cos B = \dfrac{12}{13}; B \approx 22.6°$

73. $\sin \theta = \dfrac{X_L}{Z}; \sin 24.2° = \dfrac{22.6}{Z}$

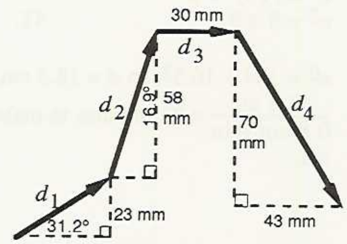
    $Z = \dfrac{22.6}{\sin 24.2°} \approx 55.1$ ohms
77. $E = \dfrac{45}{2.5 \cos 15°} \approx 18.6$ volts
81. $\sin 31.2° = \dfrac{23}{d_1}; \; d_1 = \dfrac{23}{\sin 31.2°} \approx$ 44.399 mm

    $\cos 16.9° = \dfrac{58}{d_2}; \; d_2 = \dfrac{58}{\cos 16.9°} \approx$ 60.618 mm

    $d_3 = 30$ mm

    $d_4 = \sqrt{70^2 + 43^2} \approx 82.152$ mm

    distance $= d_1 + d_2 + d_3 + d_4 = 217.2$ mm

85. $a = \sqrt{122^2 + 83^2} \approx 147.5567687$ cm

    $\tan \theta_1 = \dfrac{83}{122}; \theta_1 \approx 34.22854584°$

    $\theta_2 = \theta_1 + 5° \approx 39.22854584°$

    $\theta_3 = \theta_2 + 5° \approx 44.22854584°$

    $\cos \theta_2 = \dfrac{a}{c}; \; c = \dfrac{a}{\cos \theta_2} =$

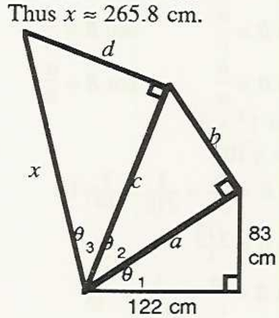
    $\dfrac{147.5567687}{\cos 39.22854584°} \approx 190.4868947$

    cm; $\cos \theta_3 = \dfrac{c}{x}; x = \dfrac{c}{\cos \theta_3}$

    $= \dfrac{190.4868947}{\cos 44.22854584°}$

    ≈ 265.8340543 cm

    Thus $x \approx 265.8$ cm.

Exercise 1-4

1. $\tan \theta \cot \theta$

   $\dfrac{1}{\cot \theta} \cdot \cot \theta$

   1
5. $\sec \theta (\cot \theta + \cos \theta - 1)$

   $\sec \theta \cdot \cot \theta + \sec \theta \cdot \cos \theta - \sec \theta$

   $\dfrac{1}{\cos \theta} \cdot \dfrac{\cos \theta}{\sin \theta} + \dfrac{1}{\cos \theta} \cdot \cos \theta - \sec \theta$

   $\dfrac{1}{\sin \theta} + 1 - \sec \theta$

   $\csc \theta + 1 - \sec \theta$
9. $1 - \cos^2\theta$

   $(\sin^2\theta + \cos^2\theta) - \cos^2\theta$

   $\sin^2\theta$
13. $(\cos \theta + \sin \theta)(\cos \theta - \sin \theta) + 2 \sin^2\theta$

    $\cos^2\theta - \cos \theta \cdot \sin \theta + \sin \theta \cos \theta$

    $- \sin^2 \theta + 2 \sin^2\theta$

    $\cos^2\theta + \sin^2\theta$

    1
17. | $\tan 32°40'$ | $\dfrac{\sin 32°40'}{\cos 32°40'}$ |
    |---|---|
    | 0.64117 | $\dfrac{0.53975}{0.84182} \approx 0.64117$ |

21. $\sin\beta(\cot\beta - \csc\beta + \sin\beta)$
$\sin\beta \cdot \cot\beta - \sin\beta \cdot \csc\beta + \sin\beta \cdot \sin\beta$
$\sin\beta \cdot \dfrac{\cos\beta}{\sin\beta} - \sin\beta \cdot \dfrac{1}{\sin\beta} + \sin^2\beta$
$\cos\beta - 1 + \sin^2\beta$
$\cos\beta - (\cos^2\beta + \sin^2\beta) + \sin^2\beta$
$\cos\beta - \cos^2\beta$

25. $2\cos x = 1$
$\cos x = \frac{1}{2}$
$x = \cos^{-1}\frac{1}{2} = 60°$

29. $5\sin x = 1$
$\sin x = \frac{1}{5}$
$x = \sin^{-1}\frac{1}{5} \approx 11.5°$

33. $\dfrac{\sin x}{3} = \dfrac{2}{11}$
$\sin x = \frac{6}{11}$
$x = \sin^{-1}\frac{6}{11} \approx 33.1°$

37. $\tan 2x = \sqrt{3}$
$2x = \tan^{-1}\sqrt{3}$
$2x = 60°$
$x = 30°$

41. $2\cos 4x = 1$
$\cos 4x = \frac{1}{2}$
$4x = \cos^{-1}\frac{1}{2}$
$4x = 60°$
$x = 15°$

45. $\sin B = \dfrac{b}{c}$, $\cos B = \dfrac{a}{c}$
$\dfrac{\sin B}{\cos B} = \dfrac{\frac{b}{c}}{\frac{a}{c}} = \dfrac{b}{c} \cdot \dfrac{c}{a} = \dfrac{b}{a} =$
$\tan B$

Chapter 1 Review

1. $17°34'17'' = (17 + \frac{34}{60} + \frac{17}{3600})° \approx 17.57°$; acute

3. $125°37' = (125 + \frac{37}{60})° \approx 125.62°$; obtuse

5. $180° - (81°43'12'' + 38°19'56'') =$
$180° - 120°3'8'' = 59°56'52''$

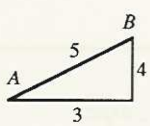
7. $c = \sqrt{a^2 + b^2} = \sqrt{8^2 + 12^2} = \sqrt{208}$
$= \sqrt{16 \cdot 13} = 4\sqrt{13} \approx 14.4$

9. $a = \sqrt{c^2 - b^2}$
$= \sqrt{13^2 - 8^2}$
$= \sqrt{105} \approx 10.2$

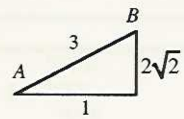
11. $R = \sqrt{Z^2 - X_L{}^2} = \sqrt{56.6^2 - 40^2}$
≈ 40.0 ohms

13. $b = \sqrt{15^2 - 5^2} = \sqrt{200} = 10\sqrt{2}$
$\sin B = \frac{b}{c} = \frac{10\sqrt{2}}{15} = \frac{2}{3}\sqrt{2}$
$\csc B = \frac{3}{2\sqrt{2}} = \frac{3}{4}\sqrt{2}$
$\cos B = \frac{a}{c} = \frac{5}{15} = \frac{1}{3}$ $\qquad \sec B = 3$
$\tan B = \frac{b}{a} = \frac{10\sqrt{2}}{5} = 2\sqrt{2}$
$\cot B = \frac{1}{2\sqrt{2}} = \frac{1}{4}\sqrt{2}$

15. $\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$



17. $\sec A = 3$ so $\cos A = \frac{1}{3}$;



$\tan B = \frac{\text{opp}}{\text{adj}}$
$= \frac{1}{2\sqrt{2}} = \frac{1}{4}\sqrt{2}$

19. $\sin 15.5° \approx 0.2672$

21. $\tan 17.9° \approx 0.3230$

23. $\tan 16.3° \approx 0.2924$

25. $\cot 31°20' = \dfrac{1}{\tan 31°20'} \approx 1.6426$
$\boxed{(}$ 31 $\boxed{+}$ 20 $\boxed{\div}$ 60 $\boxed{)}$ $\boxed{\text{TAN}}$ $\boxed{1/x}$
$\boxed{\text{TI-81}}$ $\boxed{(}$ $\boxed{\text{TAN}}$ $\boxed{(}$ 31 $\boxed{+}$ 20 $\boxed{\div}$ 60
$\boxed{)}$ $\boxed{)}$ $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$

27. $\tan 31°30' \approx 0.6128$

29. $\tan 63°30' \approx 2.0057$

31. $\sin 53.2° \approx 0.8007$
$\csc 53.2° \approx 1.2489$
$\cos 53.2° \approx 0.5990$
$\sec 53.2° \approx 1.6694$
$\tan 53.2° \approx 1.3367$
$\cot 53.2° \approx 0.7481$
$\sin 53°20' \approx 0.8021$
$\csc 53°20' \approx 1.2467$
$\cos 53°20' \approx 0.5972$
$\sec 53°20' \approx 1.6746$
$\tan 53°20' \approx 1.3432$
$\cot 53°20' \approx 0.7445$

33. $\theta = \sin^{-1} 0.6314 \approx 39.2°$

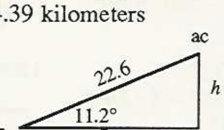
35. $\theta = \tan^{-1}(\frac{35.9}{50.0}) \approx 35.7°$

37. $\sin\theta = \frac{1}{1.425}$; $\theta = \sin^{-1}\frac{1}{1.425} \approx 44.6°$

39. $B = 90° - 56.1° = 33.9°$
$\sin 56.1° = \dfrac{23.3}{c}$; $c = \dfrac{23.3}{\sin 56.1°} \approx 28.1$
$\tan 56.1° = \dfrac{23.3}{b}$; $b = \dfrac{23.3}{\tan 56.1°} \approx 15.7$

41. $c = \sqrt{4^2 + 8.55^2} \approx 9.44$
$\tan A = \frac{4}{8.55}$; $A = \tan^{-1}\frac{4}{8.55} \approx 25.07°$
$B \approx 90° - 25.07° = 64.93°$

43. $\sin 11.2° = \dfrac{h}{22.6}$; $h = 22.6 \sin 11.2°$
≈ 4.39 kilometers



45. $\sin^2 45° + \cos^2 45°$
$(\frac{\sqrt{2}}{2})^2 + (\frac{\sqrt{2}}{2})^2$
$\frac{2}{4} + \frac{2}{4} = 1$

47. $\csc\theta(\sin\theta - \tan\theta)$
$\dfrac{1}{\sin\theta} \cdot \sin\theta - \dfrac{1}{\sin\theta} \cdot \dfrac{\sin\theta}{\cos\theta}$
$1 - \dfrac{1}{\cos\theta}$
$1 - \sec\theta$

49. $\dfrac{\cos\alpha + 2 - \cot\alpha}{\cos\alpha}$
$\dfrac{\cos\alpha}{\cos\alpha} + \dfrac{2}{\cos\alpha} - \dfrac{\cot\alpha}{\cos\alpha}$
$1 + 2\sec\alpha - \dfrac{\cos\alpha}{\sin\alpha} \cdot \dfrac{1}{\cos\alpha}$
$1 + 2\sec\alpha - \dfrac{1}{\sin\alpha}$
$1 + 2\sec\alpha - \csc\alpha$

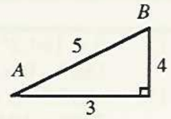
51. $3\sin 2x = 2$
$\sin 2x = \frac{2}{3}$
$2x = \sin^{-1}\frac{2}{3}$
$x = \frac{1}{2}\sin^{-1}\frac{2}{3} \approx 20.9°$

Chapter 1 Test

1. $(26 + \frac{27}{60} + \frac{43}{3600})° \approx 26.462°$

3. $b = \sqrt{12^2 - 6^2} = \sqrt{108} = \sqrt{36 \cdot 3}$
$= 6\sqrt{3}$

5. $\sec A = \frac{5}{3}$, so $\cos A = \frac{3}{5}$; $\tan B = \frac{3}{4}$



7. $\sec 78.3° = \dfrac{1}{\cos 78.3°} \approx 4.9313$

9. $P = 220 \cdot 4.1 \cos 48° \approx 603.6$ watts

11. $\cos\theta = \frac{1}{4.121}$; $\theta = \cos^{-1}\frac{1}{4.121} \approx 76.0°$

13. $c = \sqrt{5.2^2 + 7.9^2} \approx 9.5$
$\tan A = \frac{5.2}{7.9}$; $A = \tan^{-1}\frac{5.2}{7.9} \approx 33.4°$
$B \approx 90° - 33.4° = 56.6°$

15. $\cos\theta(\sec\theta - \cos\theta)$
$\cos\theta \cdot \dfrac{1}{\cos\theta} - \cos^2\theta$
$1 - \cos^2\theta$
$\sin^2\theta$

Chapter 2

Exercise 2-1

1. a. Is a function because no first element repeats.
   b. domain is first elements: {3,4,6,7} range is second elements: {5,9,10}
   c. Not one to one because a second element (5) repeats.

5. a. Not a function because a first element (1) repeats.

9. a. 9   b. 19   c. 11
   d. f(7) is not defined because 7 is not one of the first elements of any of the ordered pairs in f. d. f(250) not defined for the same reasons.

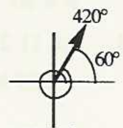
13. $h(x) = (x-3)(x+2)$
   a. $h(-2) = (-2-3)(-2+2) = 0$ $(-2,0)$
   b. $h(0) = (0-3)(0+2) = -6$ $(0,-6)$
   c. $h(\sqrt{3}) = (\sqrt{3}-3)(\sqrt{3}+2) = 3 + 2\sqrt{3} - 3\sqrt{3} - 6 = -3 - \sqrt{3}$ $(\sqrt{3}, -3-\sqrt{3})$
   d. $h(\frac{1}{2}) = (\frac{1}{2} - 3)(\frac{1}{2} + 2) = (-\frac{5}{2})(\frac{5}{2}) = -\frac{25}{4}$ $(\frac{1}{2}, -\frac{25}{4})$

17. $f(x) = 3x^4 - x^2 + 2$
   a. $f(-2) = 3(-2)^4 - (-2)^2 + 2 = 46$ $(-2,46)$
   b. $f(0) = 3 \cdot 0^4 - 0^2 + 2 = 2$ $(0,2)$
   c. $f(\sqrt{3}) = 3(\sqrt{3})^4 - (\sqrt{3})^2 + 2 = 26$ $(\sqrt{3},26)$
   d. $f(\frac{1}{2}) = 3(\frac{1}{2})^4 - (\frac{1}{2})^2 + 2 = \frac{31}{16}$ $(\frac{1}{2}, \frac{31}{16})$

21. $t(x) = 500 - 2x$
   a. $t(3) = 500 - 2(3) = 494$C°
   b. $t(12) = 500 - 2(12) = 476$C°
   c. $t(104) = 500 - 2(104) = 292$C°

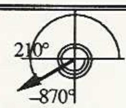
Exercise 2–2

1. 420°
   $420° - 360° = 60°$

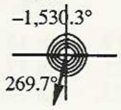


5. 1,800.6°
   $1{,}800.6° - 5 \cdot 360° = 0.6°$

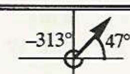
9. –870°
   $-870° + 3 \cdot 360° = 210°$



13. $-1{,}530.3° + 5 \cdot 360° = 269.7°$



17. –313°
   $-313° + 360° = 47°$



21. $-6.1° + 360° = 353.9°$ ATDC

25. $(-5, 8)$ $r = \sqrt{(-5)^2 + 8^2} = \sqrt{89}$

$\sin\theta = \frac{8}{\sqrt{89}} = \frac{8\sqrt{89}}{89}$ $\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{89}}{8}$

$\cos\theta = \frac{-5}{\sqrt{89}} = \frac{-5\sqrt{89}}{89}$ $\sec\theta = \frac{1}{\cos\theta} = -\frac{\sqrt{89}}{5}$

$\tan\theta = \frac{8}{-5} = -1\frac{3}{5}$ $\cot\theta = \frac{1}{\tan\theta} = -\frac{5}{8}$

29. $(-1, 4)$ $r = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$

$\sin\theta = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$ $\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{17}}{4}$

$\cos\theta = \frac{-1}{\sqrt{17}} = -\frac{\sqrt{17}}{17}$ $\sec\theta = \frac{1}{\cos\theta} = -\sqrt{17}$

$\tan\theta = \frac{4}{-1} = -4$ $\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{4}$

33. $(-\sqrt{2}, 6)$ $r = \sqrt{(-\sqrt{2})^2 + 6^2} = \sqrt{38}$

$\sin\theta = \frac{y}{r} = \frac{6}{\sqrt{38}} = \frac{6\sqrt{38}}{38} = \frac{3\sqrt{38}}{19}$ $\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{38}}{6}$

$\cos\theta = \frac{x}{r} = \frac{-\sqrt{2}}{\sqrt{38}} = -\frac{\sqrt{76}}{38} = -\frac{2\sqrt{19}}{38} = -\frac{\sqrt{19}}{19}$ $\sec\theta = \frac{1}{\cos\theta} = -\frac{\sqrt{38}}{\sqrt{2}} = -\frac{\sqrt{76}}{2} = -\frac{2\sqrt{19}}{2} = -\sqrt{19}$

$\tan\theta = \frac{y}{x} = \frac{6}{-\sqrt{2}} = -\frac{6\sqrt{2}}{2} = -3\sqrt{2}$ $\cot\theta = \frac{1}{\tan\theta} = -\frac{\sqrt{2}}{6}$

37. $(\sqrt{6}, -\sqrt{10})$ $r = \sqrt{(\sqrt{6})^2 + (-\sqrt{10})^2} = \sqrt{16} = 4$

$\sin\theta = -\frac{\sqrt{10}}{4}$ $\csc\theta = -\frac{4}{\sqrt{10}} = -\frac{4\sqrt{10}}{10} = -\frac{2\sqrt{10}}{5}$

$\cos\theta = \frac{\sqrt{6}}{4}$ $\sec\theta = \frac{1}{\cos\theta} = \frac{4}{\sqrt{6}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$

$\tan\theta = -\frac{\sqrt{10}}{\sqrt{6}} = -\frac{\sqrt{60}}{6} = -\frac{2\sqrt{15}}{6} = -\frac{\sqrt{15}}{3}$ $\cot\theta = \frac{1}{\tan\theta} = -\frac{3}{\sqrt{15}} = -\frac{3\sqrt{15}}{15} = -\frac{\sqrt{15}}{5}$

41. $(\sqrt{2}b, b)$ $r = \sqrt{(\sqrt{2}b)^2 + b^2} = \sqrt{2b^2 + b^2} = \sqrt{3b^2} = \sqrt{3}b$

$\sin\theta = \frac{b}{\sqrt{3}b} = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$ $\csc\theta = \sqrt{3}$

$\cos\theta = \frac{\sqrt{2}b}{\sqrt{3}b} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{1}{3}\sqrt{6}$ $\sec\theta = \frac{\sqrt{3}}{\sqrt{2}} = \frac{1}{2}\sqrt{6}$

$\tan\theta = \frac{b}{\sqrt{2}b} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$ $\cot\theta = \sqrt{2}$

45. $\sec\theta = \frac{r}{x}$ Definition of sec θ.
$= \frac{1}{\frac{x}{r}} = \frac{1}{\cos\theta}$

49. $\sin\theta = \frac{y}{r} = \frac{1}{\frac{r}{y}} = \frac{1}{\csc\theta}$

Exercise 2–3

1. $\sin\theta > 0$, $\cos\theta < 0$
   I, II   II, III
   II

5. $\tan\theta < 0$, $\csc\theta < 0$
   $\sin\theta < 0$
   II, IV   III, IV
   IV

9. $\sec\theta > 0$, $\sin\theta < 0$
   $\cos\theta > 0$
   I, IV   III, IV
   IV

13. θ = 164.2° in Quadrant II, so θ' = 180° – θ = 180° – 164.2° = 15.8°.

17. –255.3° is coterminal with –255.3° + 360° = 104.7°. θ = 104.7° in Quadrant II, so θ' = 180° – θ = 180° – 104.7° = 75.3°. Thus θ' for –255.3° is 75.3°.

21. $-181.0°$ is coterminal with $-181.0° + 360° = 179.0°$. $\theta = 179.0°$ in Quadrant II, so $\theta' = 180° - \theta = 180° - 179.0° = 1.0°$. Thus θ' for $-181.0°$ is $1.0°$.

25. $-252°$ is coterminal with $-252° + 360° = 108°$. $\theta = 108°$ in Quadrant II, so $\theta' = 180° - \theta = 180° - 108° = 72°$. Thus θ' for $-252°$ is $72°$.

29. $\theta' = 30°$; $\sin 30° = \frac{1}{2}$. In QIII, so $\sin 210° < 0$: $\sin 210° = -\frac{1}{2}$.

33. $\sin(-120°)$ $\theta' = 60°$; $\sin 60° = \frac{\sqrt{3}}{2}$. In QIII, so $\sin(-120°) < 0$: $\sin(-120°) = -\frac{\sqrt{3}}{2}$

37. $\theta' = 60°$; $\cot 60° = \dfrac{1}{\tan 60°} = \dfrac{\sqrt{3}}{3}$. $300°$ is in Quadrant IV, where cotangent is negative, so $\cot 300° = -\frac{\sqrt{3}}{3}$.

41. $\csc 90° = \dfrac{1}{\sin 90°} = 1$

45. $\theta' = 30°$. $\cos 30° = \frac{\sqrt{3}}{2}$; $150°$ in Quadrant II, so $\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$. $\sec 150° = \dfrac{1}{\cos 150°} = \dfrac{1}{-\frac{\sqrt{3}}{2}} = -\dfrac{2}{\sqrt{3}} = -\dfrac{2\sqrt{3}}{3}$.

49. 0.6898537919

53. 1.026304108
13 $\boxed{+/-}$ $\boxed{\cos}$ $\boxed{1/x}$
$\boxed{\text{TI-81}}$ $\boxed{(}$ $\boxed{\text{COS}}$ $\boxed{(-)}$ 13 $\boxed{)}$ $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$

57. – 1.036744916

61. $\theta = \sin^{-1}0.25 \approx 14.5°$

65. $\theta = \tan^{-1}\left(-\frac{8}{5}\right) \approx -57.995° \approx -58.0°$
8 $\boxed{\div}$ 5 $\boxed{=}$ $\boxed{\pm}$ $\boxed{\text{SHIFT}}$ $\boxed{\tan}$
$\boxed{\text{TI-81}}$ $\boxed{\text{2nd}}$ $\boxed{\text{TAN}}$ $\boxed{(}$ $\boxed{(-)}$ 8 $\boxed{\div}$ 5 $\boxed{)}$ $\boxed{\text{ENTER}}$

69. $E = 156 \sin(\theta + 45°)$

(a) $\theta = 0°$	$E = 156 \sin(0° + 45°)$	$= 156 \sin 45°$	$\approx$	110.31
(b) $\theta = 45°$	$E = 156 \sin(45° + 45°)$	$= 156 \sin 90°$	$=$	156
(c) $\theta = 100°$	$E = 156 \sin(100° + 45°)$	$= 156 \sin 145°$	$\approx$	89.48
(d) $\theta = -200°$	$E = 156 \sin(-200° + 45°)$	$= 156 \sin(-155°)$	$\approx$	–65.93
(e) $\theta = 13.3°$	$E = 156 \sin(13.3° + 45°)$	$= 156 \sin 58.3°$	$\approx$	132.73
(f) $\theta = -45°$	$E = 156 \sin(-45° + 45°)$	$= 156 \sin 0°$	$=$	0

73. $\sin(2\theta) = 2 \sin\theta$
$\sin(2 \cdot 30°) = 2 \sin 30°$ $\quad \frac{\sqrt{3}}{2} = 2 \cdot \frac{1}{2}$
$\sin 60° = 2 \sin 30°$ $\quad \frac{\sqrt{3}}{2} = 1$ No

Exercise 2–4

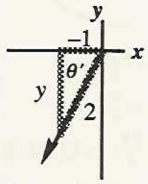
1. $\sin\theta = 0.8251$, $\cos\theta > 0$
$\sin\theta' = 0.8251$, so $\theta' \approx 55.6°$
Since $\sin\theta > 0$, $\cos\theta > 0$, θ is in QI.
In QI $\theta = \theta'$, so $\theta \approx 55.6°$.

5. $\sec\theta = -1.0642$, $\sin\theta < 0$
$\cos\theta =$ –\f(1,1.0642) so $\cos\theta' = \dfrac{1}{1.0642}$; $\theta' \approx 20.0°$.
$\cos\theta < 0$, $\sin\theta < 0$, so θ is in QIII.
$\theta = 180° + \theta' \approx 180° + 20.0° \approx 200.0°$.

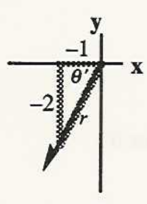
9. $\sin\theta = \frac{3}{8}$, $\cos\theta > 0$
$\sin\theta' = \frac{3}{8}$, so $\theta' \approx 22.0°$.
$\sin\theta > 0$, $\cos\theta > 0$, so θ is in QI, and
$\theta = \theta' \approx 22.0°$.

13. $\cos\theta = -\frac{5}{7}$, $\tan\theta > 0$
$\cos\theta' = \frac{5}{7}$, so $\theta' \approx 44.4°$.
$\cos\theta < 0$, $\tan\theta > 0$ so θ is in QIII.
$\theta = 180° + \theta' \approx 180° + 44.4° \approx 224.4°$.

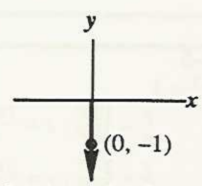
17. $\cos\theta = -\frac{1}{2}$, $\tan\theta > 0$; $\sec\theta = -2$
$\cos\theta < 0$, $\tan\theta > 0$ so θ in QIII.
$y = -\sqrt{2^2 - (-1)^2} = -\sqrt{3}$
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{y}{2} = -\dfrac{\sqrt{3}}{2}$ $\csc\theta = -\frac{2}{3}\sqrt{3}$
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{y}{-1} = \dfrac{-\sqrt{3}}{-1} = \sqrt{3}$
$\cot\theta = \frac{1}{3}\sqrt{3}$
$\cos\theta' = \frac{1}{2}$, so $\theta' = 60°$.
$\theta = 180° + \theta' = 180° + 60° = 240°$.



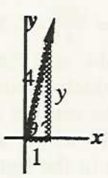
21. $\tan\theta = 2$, $\cos\theta < 0$; $\cot\theta = \frac{1}{2}$
$\tan\theta > 0$, $\cos\theta < 0$ so θ in QIII.
$r = +\sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{-2}{r} = -\dfrac{2}{\sqrt{5}} = -\dfrac{2\sqrt{5}}{5}$
$\csc\theta = -\frac{1}{2}\sqrt{5}$
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{-1}{r} = -\dfrac{1}{\sqrt{5}} = -\dfrac{\sqrt{5}}{5}$
$\sec\theta = -\sqrt{5}$
$\tan\theta' = 2$, so $\theta' \approx 63.4°$.
$\theta = 180° + \theta' \approx 180° + 63.4° \approx 243.4°$.



25. $\csc\theta = -1$; $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{-1} = -1$;
θ is $270°$; pick a point, say $(0, -1)$ on the terminal side of the angle.
$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-1)^2} = 1$
$\cos\theta = \dfrac{x}{r} = \dfrac{0}{1} = 0$; $\sec\theta$ undefined
$\tan\theta = \dfrac{y}{x} = \dfrac{-1}{0}$ (undefined); $\cot\theta = 0$



29. $\sec\theta = 4$, $\csc\theta > 0$
$\cos\theta = \frac{1}{4}$, $\sin\theta > 0$
$\cos\theta > 0$, $\sin\theta > 0$ so θ in QI.
$y = +\sqrt{4^2 - 1^2} = \sqrt{15}$
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{y}{4} = \dfrac{\sqrt{15}}{4}$; $\csc\theta = \frac{4}{15}\sqrt{15}$
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{y}{1} = \sqrt{15}$; $\cot\theta = \frac{1}{15}\sqrt{15}$
$\cos\theta = \frac{1}{4}$, so $\theta \approx 75.5°$
$\theta = \theta' \approx 75.5°$



33. $\cos\theta = -\frac{5}{13}$, $\sin\theta > 0$

$\sec\theta = -\frac{13}{5}$

$\cos\theta < 0$, $\sin\theta > 0$, so θ in QII.

$y = +\sqrt{13^2 - (-5)^2} = 12$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{13} = \frac{12}{13}$; $\csc\theta = \frac{13}{12}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{-5} = -\frac{12}{5}$; $\cot\theta = -\frac{5}{12}$

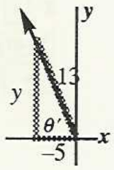
$\cos\theta' = \frac{5}{13}$

so $\theta' \approx 67.4°$.

$\theta = 180° - \theta'$

$\approx 180° - 67.4°$

$\approx 112.6°$



37. $\sec\theta = 5$, $\tan\theta > 0$

$\cos\theta = \frac{1}{5}$, $\tan\theta > 0$

$\cos\theta > 0$, $\tan\theta > 0$ so θ in QI.

$y = +\sqrt{5^2 - 1^2} = 2\sqrt{6}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{5} = \frac{2\sqrt{6}}{5}$

$\csc\theta = \frac{5}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{12} = \frac{5}{12}\sqrt{6}$

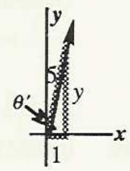
$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{1} = y = 2\sqrt{6}$

$\cot\theta = \frac{1}{2\sqrt{6}} = \frac{1}{12}\sqrt{6}$

$\cos\theta' = \frac{1}{5}$ so θ'

$\approx 78.5°$.

$\theta = \theta' \approx 78.5°$



41. $(2,-5)$; $r = \sqrt{x^2 + y^2} = \sqrt{4 + 25}$

$= \sqrt{29}$

$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{29}} = \frac{-5\sqrt{29}}{29}$

$\csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{29}}{5}$

$\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$

$\sec\theta = \frac{\sqrt{29}}{2}$

$\tan\theta = \frac{y}{x} = -\frac{5}{2}$; $\cot\theta = -\frac{2}{5}$

$\tan\theta' = 2.5$, $\theta' \approx 68.2°$

$\sin\theta < 0$, $\cos\theta > 0$ so θ terminates in QIV. $\theta = 360° - \theta' \approx 360° - 68.2° = 291.8°$

45. $\theta = -134.4°$, $r = 8.25$.

$\sin\theta = \frac{y}{r}$, so $y = r\sin\theta$, so $y = 8.25\sin(-134.4°) \approx -5.89$ cm.

$\cos\theta = \frac{x}{r}$, so $x = r\cos\theta$, so $x = 8.25$ $\cos(-134.4°) \approx -5.77$ cm

49. $4'\,3.5'' = 4\frac{3.5}{12}'' \approx 4.292'$; $r \approx \frac{4.292}{2}$ ft ≈ 2.146 ft;

$\theta = 211.5°$. $\sin\theta = \frac{y}{r}$; $y = r\sin\theta$;

$y \approx 2.146 \sin 211.5° \approx -1.121$ ft.

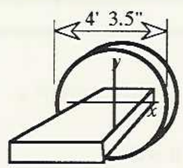
0.121 ft x 12"/ft ≈ 1.5", so $y \approx -1'\,1.5''$

$\cos\theta = \frac{x}{r}$; $x = r\cos\theta$

$x \approx 2.146\cos 211.5° \approx -1.830$ ft.

0.830 ft x 12"/ft ≈ 10.0", so

$x \approx -1'\,10.0''$



53. $\tan\theta = u$ and θ terminates in quadrant III.

Note that we use the values $\frac{-u}{-1}$ to represent the value u. This is because the side adjacent to θ' must be negative in quadrant III.

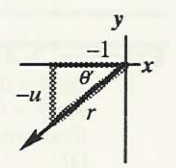
$r = \sqrt{1^2 + u^2} = \sqrt{1 + u^2}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{u}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{-u}{r} = -\frac{u}{\sqrt{1 + u^2}}$ $\qquad \csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{1 + u^2}}{u}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{-1}{r} = -\frac{1}{\sqrt{1 + u^2}}$

$\sec\theta = \frac{1}{\cos\theta} = -\sqrt{1 + u^2}$



57. $\sin p = \frac{AB \sin b}{AP} = \frac{512.4 \cdot \sin 28.3°}{322.6} \approx 0.75302$;

$p \approx 48.852°$

$a = 180° - (b + p) \approx 02.848°$

$BP = \frac{AP \sin a}{\sin b} \approx \frac{322.6 \cdot \sin 102.848°}{\sin 28.3°} \approx 663.4$ ft.

Exercise 2–5

1. $x^2 + y^2 = 1$

5. 100° $\quad \frac{s}{\pi} = \frac{100°}{180°}$; $s = \frac{100° \cdot \pi}{180°} = \frac{5\pi}{9} \approx 1.75$

9. 270° $\quad \frac{s}{\pi} = \frac{270°}{180°}$; $s = \frac{270° \cdot \pi}{180°} = \frac{3\pi}{2} \approx 4.71$

13. –305° $\quad \frac{s}{\pi} = \frac{-305°}{180°}$; $s = \frac{-305° \cdot \pi}{180°} = -\frac{61\pi}{36}$

≈ -5.32

In problems 14 through 30 we solve the definition of radians for $\theta°$:

$\frac{s}{\pi} = \frac{\theta°}{180°}$; $\boxed{\theta° = \frac{180°}{\pi} \cdot s}$.

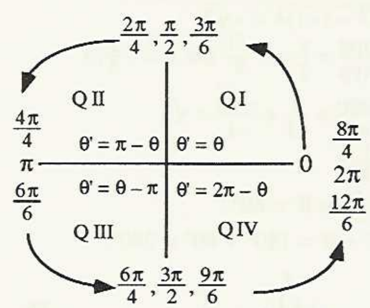
17. $\frac{3\pi}{5}$ $\quad \theta° = \frac{180°}{\pi} \cdot \frac{3\pi}{5} = 108°$

21. $-\frac{17\pi}{6}$ $\quad \theta° = \frac{180°}{\pi} \cdot (-\frac{17\pi}{6}) = -510°$

25. $-\frac{12}{17}$ $\quad \theta° = \frac{180°}{\pi} \cdot (-\frac{12}{17}) = -\frac{2160°}{17\pi} \approx -40.44°$

29. -5 $\quad \theta° = \frac{180°}{\pi} \cdot (-5) = -\frac{900}{\pi} \approx -286.48°$

In problems 31–44 add 2π to negative values until there is a coterminal angle which is positive. Then change each denominator of 3 to 6 to be able to use the figure. Then compare the expression to the figure to find in which quadrant the angle terminates. Then use the formula shown in the figure to find θ'.



33. $\frac{11\pi}{6}$; in Q IV; $\theta' = 2\pi - \theta = \frac{12\pi}{6} - \frac{11\pi}{6} = \frac{\pi}{6}$

37. $\frac{3\pi}{4}$; in Q II; $\theta' = \pi - \theta = \frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}$

41. $-\frac{2\pi}{3} + 2\pi = -\frac{2p}{3} + \frac{6p}{3} = \frac{4p}{3}$; $\frac{4\pi}{3} = \frac{8\pi}{6}$; in Q III; $\theta' = \theta - \pi$

$= \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$

45. See the left side of figure 2.14.

49. $r = 4.5$ mm, $L = 12$ mm;

$L = rs$; $12 = 4.5s$; $s \approx 2.7$ radians

53. Find L where $r = \frac{32.4}{2} = 16.2$ inches and

$\theta° = 85°$.

$85° = \frac{17}{36}\pi$

$L = rs$

$L = 16.2 \cdot \frac{17\pi}{36} \approx 24.0$ in

57. $Ap = \frac{sr^2}{2} = \frac{\frac{3\pi}{5} \cdot 6^2}{2} = 18 \cdot \frac{3\pi}{5} = \frac{54}{5}\pi \approx 33.93\ \text{cm}^2$

61. $Ap = \frac{\theta°(\pi r^2)}{360°} = \frac{15° \cdot \pi \cdot 9^2}{360°} = \frac{27}{8}\pi \approx 10.60\ \text{mm}^2$

63. $Ap = \frac{sr^2}{2}$

$14.6 = \frac{s \cdot 4.852^2}{2}$

$29.2 = 4.852^2 s$

$s = \frac{29.2}{4.852^2} \approx 1.24$ radians

64. $Ap = \frac{\theta°(\pi r^2)}{360°}$; $r = 50/2 = 25$ in

$200 = \frac{\theta°(\pi \cdot 25^2)}{360°}$

$200 = \frac{625\pi}{360} \cdot \theta°$

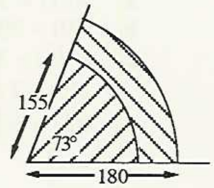
$\theta° = \frac{200 \cdot 360}{625\ \pi} = \frac{576}{5\pi} \approx 36.7°$

65. The new area to be covered is the area of the larger, outer sector less the inner sector.

$\frac{73° \cdot \pi \cdot 180^2}{360°} - \frac{73° \cdot \pi \cdot 155^2}{360°} \approx 5335.3\ \text{ft}^2$

Dividing this by 150 ft^2 per gallon gives $\frac{5335.3}{150} \approx 35.57$ gallons, which is $35\frac{1}{2}$ gallons to the nearest half gallon.

Note: The calculation can be simplified algebraically to $\frac{73\pi}{360}(180^2 - 155^2)$.



Exercise 2–6

1. $\frac{2\pi}{3}$ is in quadrant II, so $\sin\frac{2\pi}{3} > 0$ and $\theta' = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ so $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$.

5. $\frac{4\pi}{3}$ is in quadrant III, so $\cos\frac{4\pi}{3} < 0$ and $\theta' = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$. $\cos\frac{\pi}{3} = \frac{1}{2}$, so $\cos\frac{4\pi}{3} = -\frac{1}{2}$.

9. $\frac{5\pi}{3}$ is in quadrant IV, so $\tan\frac{5\pi}{3} < 0$ and $\theta' = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$. $\tan\frac{\pi}{3} = \sqrt{3}$, so $\tan\frac{5\pi}{3} = -\sqrt{3}$.

13. $-\frac{7\pi}{6}$ is coterminal with $\frac{5\pi}{6}$ so $\sin(-\frac{7\pi}{6}) = \sin\frac{5\pi}{6} = \frac{1}{2}$

Make sure the calculator is in radian mode.

17. $\tan 0.5 \approx 0.5463$

21. $\sin 2.3 \approx 0.7457$

25. $\csc 2.5 = \frac{1}{\sin 2.5} \approx 1.6709$

29. $\theta' = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

$\sin\theta < 0$, $\tan\theta < 0$ so θ in qIV

$\theta = 2\pi - \theta' = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

33. $\sin\theta = -\frac{1}{2}$ so $\theta' = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$\sin\theta < 0$, $\cos\theta > 0$ so θ in qIV.

$\theta = 2\pi - \theta' = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

37. $\theta' = \cos^{-1}0.885 \approx 0.484$

$\cos\theta < 0$, $\tan\theta > 0$ so θ in qIII

$\theta = \pi + \theta' \approx \pi + 0.484 \approx 3.63$

41. $3\sin 2\theta = 1$

$\sin 2\theta = \frac{1}{3}$

$2\theta = \sin^{-1}\frac{1}{3}$

$\theta = \frac{1}{2}\sin^{-1}\frac{1}{3} \approx 0.17$

3 $\boxed{1/x}$ $\boxed{\sin^{-1}}$ $\boxed{\div}$ 2 $\boxed{=}$

TI–81: $\boxed{(}$ 1 $\boxed{\div}$ 2 $\boxed{)}$ $\boxed{\text{2nd}}$ $\boxed{\sin^{-1}}$ $\boxed{(}$ 1 $\boxed{\div}$ 3 $\boxed{)}$

Note: 3 $\boxed{x^{-1}}$ is the same as 1 $\boxed{\div}$ 3

45. $\sin\frac{\theta}{2} = 1$

$\frac{\theta}{2} = \sin^{-1}1$

$\frac{\theta}{2} = \frac{\pi}{2}$

$\theta = 2(\frac{\pi}{2})$

$\theta = \pi$

49. $\cos\theta(2\cos\theta - 1) = 0$

$\cos\theta = 0$ or $2\cos\theta - 1 = 0$

$\theta = \cos^{-1}0$ or $2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}\frac{1}{2}$

$\theta = \frac{\pi}{2}$ or $\frac{\pi}{3}$

53. $d = \frac{1}{3}\cos 8t - \frac{1}{4}\sin 8t$

(a) $t = \frac{1}{8} = 0.125$:

$d = \frac{1}{3}\cos 8(0.125) - \frac{1}{4}\sin 8(0.125)$

≈ -0.030

(b) $t = \frac{1}{4} = 0.25$:

$d = \frac{1}{3}\cos 8(0.25) - \frac{1}{4}\sin 8(0.25)$

≈ -0.366

57. $\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

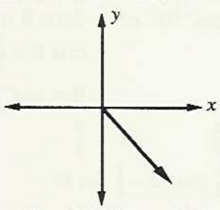
			Using Calculator
(a) $x = 0.1$:	$\sin 0.1 \approx 0.1 - \frac{0.1^3}{6} + \frac{0.1^5}{120} - \frac{0.1^7}{5040}$	≈ 0.0998334166	0.0998334166
(b) $x = 0.5$:	$\sin 0.5 \approx 0.5 - \frac{0.5^3}{6} + \frac{0.5^5}{120} - \frac{0.5^7}{5040}$	≈ 0.4794255332	0.4794255386
(c) $x = 1$:	$\sin 1 \approx 1 - \frac{1^3}{6} + \frac{1^5}{120} - \frac{1^7}{5040}$	≈ 0.8414682540	0.8414709848
(d) $x = (\frac{\pi}{6})$:	$\sin\frac{\pi}{6} \approx \frac{\pi}{6} - \frac{(\frac{\pi}{6})^3}{6} + \frac{(\frac{\pi}{6})^5}{120} - \frac{(\frac{\pi}{6})^7}{5040}$	≈ 0.4999999919	0.5

Note: To calculate part (d) most easily, compute $\frac{\pi}{6}$ and store it in the calculator's memory. Assuming $\boxed{\text{Min}}$ is the key to enter a value into memory, and $\boxed{\text{MRec}}$ is the key to recall the value in memory, then the following will calculate part d: $\boxed{\pi}$ $\boxed{\div}$ 6 $\boxed{=}$ $\boxed{\text{Min}}$ $\boxed{-}$ $\boxed{\text{MRec}}$ $\boxed{x^y}$ 3 $\boxed{\div}$ 6 $\boxed{+}$ $\boxed{\text{MRec}}$ $\boxed{x^y}$ 5 $\boxed{\div}$ 120 $\boxed{-}$ $\boxed{\text{MRec}}$ $\boxed{x^y}$ 7 $\boxed{\div}$ 5040 $\boxed{=}$

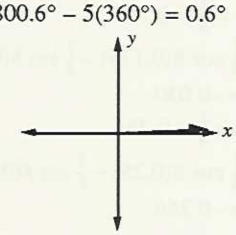
On the TI–81, store the equation as Y_1: $\boxed{\text{Y=}}$ $\boxed{\text{CLEAR}}$ $\boxed{\text{X|T}}$ $\boxed{-}$ $\boxed{\text{X|T}}$ $\boxed{\text{MATH}}$ 3 $\boxed{\div}$ 6 $\boxed{+}$ $\boxed{\text{X|T}}$ $\boxed{\wedge}$ 5 $\boxed{\div}$ 120 $\boxed{-}$ $\boxed{\text{X|T}}$ $\boxed{\wedge}$ 7 $\boxed{\div}$ 5040 $\boxed{\text{2nd}}$ $\boxed{\text{CLEAR}}$. To compute the value for say $x = -.1$, store this in X and calculate, as follows: 0.1 $\boxed{\text{STO}\triangleright}$ $\boxed{\text{X|T}}$ $\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ 1 $\boxed{\text{ENTER}}$

Chapter 2 Review

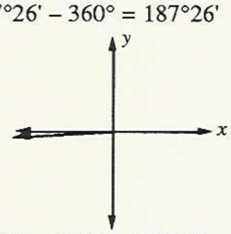
1. a. function because no first element repeats
   b. domain: {1,4,6,7}
   range: {5,7,10}
   c. Not one to one because a second element (7) repeats.
3. Not a function because a first element (2) repeats.
5. $f(x) = 3x^2 - 2x + 3$
   a. $f(-1) = 3(-1)^2 - 2(-1) + 3 = 8$
   b. $f(0) = 3(0^2) - 2(0) + 3 = 3$
   c. $f(\sqrt{5}) = 3(\sqrt{5})^2 - 2\sqrt{5} + 3 =$
   $3(5) - 2\sqrt{5} + 3 = 18 - 2\sqrt{5}$
   d. $f(\frac{1}{3}) = 3(\frac{1}{3})^2 - 2(\frac{1}{3}) + 3 = \frac{1}{3} - \frac{2}{3} + 3$
   $= 2\frac{2}{3}$
7. $f(x) = \dfrac{3x}{x-1}$
   a. $f(-1) = \dfrac{3(-1)}{-1-1} = \dfrac{-3}{-2} = \dfrac{3}{2}$
   b. $f(0) = \dfrac{3(0)}{0-1} = 0$
   c. $f(\sqrt{5}) = \dfrac{3\sqrt{5}}{\sqrt{5}-1}$
   d. $f(\frac{1}{3}) = \dfrac{3(\frac{1}{3})}{\frac{1}{3}-1} = \dfrac{1}{-\frac{2}{3}} = -\frac{3}{2}$
9. $-40.25° + 360° = 319.75°$



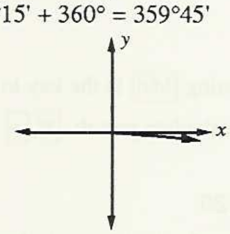
11. $1{,}800.6° - 5(360°) = 0.6°$



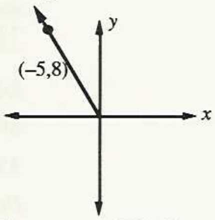
13. $547°26' - 360° = 187°26'$



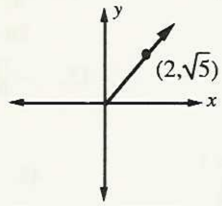
15. $-0°15' + 360° = 359°45'$



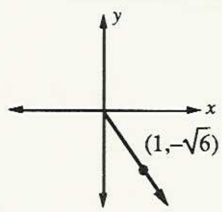
17. $(-5,8)$ $r = \sqrt{25+64} = \sqrt{89}$
   $\sin\theta = \dfrac{8}{\sqrt{89}} = \dfrac{8}{89}\sqrt{89}$ $\csc\theta = \dfrac{\sqrt{89}}{8}$
   $\cos\theta = \dfrac{-5}{\sqrt{89}} = -\dfrac{5}{89}\sqrt{89}$ $\sec\theta = -\dfrac{\sqrt{89}}{5}$
   $\tan\theta = -\dfrac{8}{5}$ $\cot\theta = -\dfrac{5}{8}$



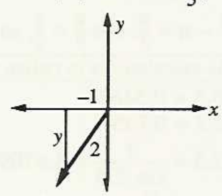
19. $(2,\sqrt{5})$ $r = \sqrt{4+5} = 3$
   $\sin\theta = \dfrac{\sqrt{5}}{3}$ $\csc\theta = \dfrac{3}{\sqrt{5}} = \dfrac{3}{5}\sqrt{5}$
   $\cos\theta = \dfrac{2}{3}$ $\sec\theta = \dfrac{3}{2}$
   $\tan\theta = \dfrac{\sqrt{5}}{2}$ $\cot\theta = \dfrac{2}{\sqrt{5}} = \dfrac{2}{5}\sqrt{5}$



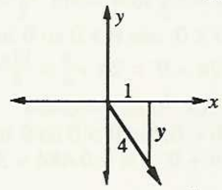
21. $(1,-\sqrt{6})$ $r = \sqrt{1+6} = \sqrt{7}$
   $\sin\theta = \dfrac{-\sqrt{6}}{\sqrt{7}} = -\dfrac{\sqrt{42}}{7}$
   $\csc\theta = -\dfrac{\sqrt{7}}{\sqrt{6}} = -\dfrac{\sqrt{42}}{6}$
   $\cos\theta = \dfrac{1}{\sqrt{7}} = \dfrac{1}{7}\sqrt{7}$ $\sec\theta = \sqrt{7}$
   $\tan\theta = \dfrac{-\sqrt{6}}{1} = -\sqrt{6}$ $\cot\theta = -\dfrac{1}{\sqrt{6}} = -\dfrac{\sqrt{6}}{6}$



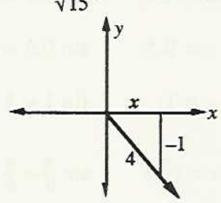
23. $\sin\theta < 0$ (III,IV), $\cos\theta < 0$ (II,III): III
25. $\cos\theta > 0$ (I,IV), $\tan\theta < 0$ (II,IV): IV
27. $\csc\theta > 0$ so $\sin\theta > 0$ (I,II), $\cot\theta < 0$ so $\tan\theta < 0$ (II,IV): II
29. $\tan\theta > 0$ (I,III), $\csc\theta < 0$ so $\sin\theta < 0$ (III,IV): III
31. $46.3°$
33. $421°48' - 360° = 61°48'$
35. $-248.7° + 360° = 111.3°$; $180° - 111.3° = 68.7°$
37. $242.57° - 180° = 62.57°$
39. $\cos 240° = -\frac{1}{2}$
41. $\csc 870° = \dfrac{1}{\sin 870°} = \dfrac{1}{\sin 150°}$
   $= \dfrac{1}{\frac{1}{2}} = 2$
43. $\tan 213.9° \approx 0.6720$
45. $\cos(-133°20') \approx -0.6862$
   133 $\boxed{+}$ 20 $\boxed{\div}$ 60 $\boxed{=}$ $\boxed{+/-}$ $\boxed{\text{COS}}$
   TI–81: $\boxed{\text{COS}}$ $\boxed{(}$ $\boxed{(-)}$ 133 $\boxed{+}$ 20 $\boxed{\div}$ 60 $\boxed{)}$ $\boxed{\text{ENTER}}$
47. $\theta' = \sin^{-1}0.3251 \approx 19.0°$;
   $\sin\theta > 0$, $\cos\theta < 0$ so θ in qII
   $\theta = 180° - \theta' \approx 180° - 19.0° = 161.0°$
49. $\theta' = \tan^{-1}0.6306 \approx 32.2°$
   θ is in qIII, $\theta \approx 180° + 32.2° = 212.2°$
51. $\cos\theta = -\dfrac{1}{2.0642}$
   $\theta' = \cos^{-1}\dfrac{1}{2.0642} \approx 61.0°$
   $\cos\theta < 0$, $\sin\theta > 0$ so θ in qII
   $\theta = 180° - \theta' \approx 180° - 61.0° = 119.0°$
53. $\theta' = \sin^{-1}\frac{\sqrt{3}}{2} = 60°$
   θ in qII so $\theta = 180° - 60° = 120°$
55. $\theta' = \cos^{-1}\frac{5}{13} \approx 67.4°$
   θ in qII so $\theta \approx 180° - 67.4° = 112.6°$
57. $\cos\theta = -\frac{1}{2}$; $\sec\theta = -2$ $y = -\sqrt{3}$
   $\sin\theta = -\frac{\sqrt{3}}{2}$; $\csc\theta = -\frac{2}{3}\sqrt{3}$
   $\tan\theta = -\sqrt{3}$; $\cot\theta = -\frac{1}{3}\sqrt{3}$



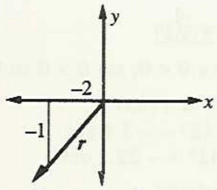
59. $\cos\theta = \frac{1}{4}$; $\sec\theta = 4$ $y = -\sqrt{15}$
   $\sin\theta = -\frac{\sqrt{15}}{4}$; $\csc\theta = -\frac{4}{15}\sqrt{15}$
   $\tan\theta = -\sqrt{15}$; $\cot\theta = -\frac{1}{15}\sqrt{15}$



61. $\csc\theta = -4$ so $\sin\theta = -\frac{1}{4}$ $x = \sqrt{15}$
   $\cos\theta = \frac{\sqrt{15}}{4}$; $\sec\theta = \frac{4}{15}\sqrt{15}$
   $\tan\theta = -\frac{1}{\sqrt{15}}$; $\cot\theta = -\sqrt{15}$



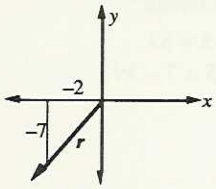
63. $\cot\theta = 2$ so $\tan\theta = \frac{1}{2}$ $r = \sqrt{5}$
   $\sin\theta = \frac{-1}{\sqrt{5}} = -\frac{1}{5}\sqrt{5}$; $\csc\theta = -\sqrt{5}$
   $\cos\theta = \frac{-2}{\sqrt{5}} = -\frac{2}{5}\sqrt{5}$; $\sec\theta = -\frac{\sqrt{5}}{2}$



65. $\tan\theta = \frac{7}{2}$, $\cot\theta = \frac{2}{7}$ $\qquad r = \sqrt{53}$

$\sin\theta = \frac{-7}{\sqrt{53}} = -\frac{7}{53}\sqrt{53}$; $\csc\theta = -\frac{1}{7}\sqrt{53}$

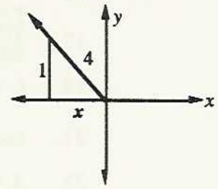
$\cos\theta = \frac{-2}{\sqrt{53}} = -\frac{2}{53}\sqrt{53}$; $\sec\theta = -\frac{1}{2}\sqrt{53}$



67. $\sin\theta = \frac{1}{4}$; $\csc\theta = 4 \qquad x = -\sqrt{15}$

$\cos\theta = \frac{-\sqrt{15}}{4}$; $\sec\theta = -\frac{4}{\sqrt{15}} = -\frac{4}{15}\sqrt{15}$

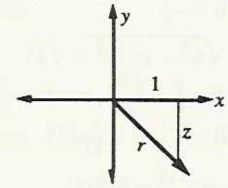
$\tan\theta = -\frac{1}{\sqrt{15}} = -\frac{1}{15}\sqrt{15}$; $\cot\theta = -\sqrt{15}$



69. The triangle represents the reference triangle for an angle in qIV whose sine is z (z must represent a negative quantity).

$r = \sqrt{1 + z^2}$. $\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$= \frac{1}{r} = \frac{1}{\sqrt{1 + z^2}}$.



71. $\sin p = \dfrac{211.5 \sin 29.6°}{185.7} \approx 0.5626$ so $p \approx 34.234°$

$a \approx 180° - (29.6° + 34.234°) = 116.166°$

$BP = \dfrac{185.7 \sin 116.166°}{\sin 29.6°} \approx 337.4$ meters

73. $x = r\cos\theta = 8.075 \cos 10.2° \approx 7.9$ inches

$y = r\sin\theta = 8.075 \sin 10.2° \approx 1.4$ inches

75. $\frac{120°}{180°} = \frac{s}{\pi}$; $\frac{2}{3} = \frac{s}{\pi}$; $s = \frac{2\pi}{3} \approx 2.09$

77. $\frac{430°}{180°} = \frac{s}{\pi}$; $s = \frac{43\pi}{18} \approx 7.50$

79. $\dfrac{\theta°}{180°} = \dfrac{\frac{11\pi}{3}}{\pi}$; $\theta° = 180° \cdot \frac{11\pi}{3} \cdot \frac{1}{\pi} = 660°$

81. $\frac{\theta°}{180°} = \frac{2.5}{\pi}$; $\theta° = 180° \cdot \frac{2.5}{\pi} = \frac{450°}{\pi} \approx 143.24°$

83. $L = rs$; $L = 3.9 \cdot \frac{6.3}{2} = 12.3$ inches

85. $150°$ is $\frac{5\pi}{6}$ radians

$L = rs = \frac{15}{2} \cdot \frac{5\pi}{6} = (6\frac{1}{4})\pi$ mm

87. $A = \frac{1}{2}sr^2$; $30°$ is $\frac{\pi}{6}$ radians

$A = \frac{1}{2} \cdot \frac{\pi}{6} \cdot 9^2 = \frac{27\pi}{4} \approx 21.21 \text{ in}^2$

89. $A = \frac{1}{2} \cdot \frac{11}{12} \cdot 6^2 = 16\frac{1}{2} \text{ mm}^2$

91. $\sin 1.9 \approx 0.9463$

93. $\tan 4.5 \approx 4.6373$

95. $\theta' = \frac{\pi}{4}$; θ in qII where $\tan\theta < 0$ so

$\tan\frac{3\pi}{4} = -\tan\frac{\pi}{4} = -1$

97. $\theta' = \frac{\pi}{6}$; θ in qIV where $\cos\theta > 0$;

$\cos(-\frac{\pi}{6}) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

99. $\theta' = \frac{\pi}{6}$; θ in qIII where $\sin\theta < 0$;

$\sin(-\frac{5\pi}{6}) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

101. $\theta' = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$; $\sin\theta < 0$, $\cos\theta > 0$ so θ in qIV;

$\theta = 2\pi - \theta' = \frac{11\pi}{6}$

103. $2\sin\theta = 0.84$

$\sin\theta = 0.42$

$\theta = \sin^{-1} 0.42 \approx 0.43$

105. $(2\cos\theta - 1)(\cos\theta - 1) = 0$

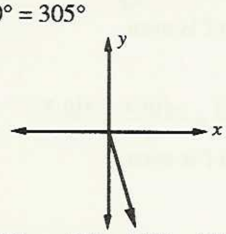
$2\cos\theta = 1 \quad$ or $\quad \cos\theta = 1$

$\cos\theta = \frac{1}{2}$

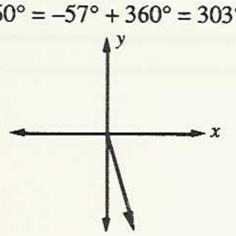
$\theta = \frac{\pi}{3} \quad$ or $\quad \theta = 0$

Chapter 2 Test

1. a. $665° - 360° = 305°$



b. $-417° + 360° = -57° + 360° = 303°$

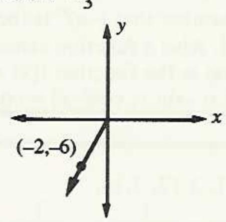


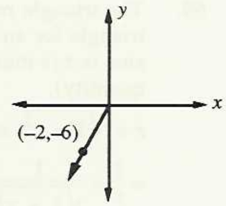
3. $(-2,-6) \qquad r = \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10}$

$\sin\theta = \frac{-6}{2\sqrt{10}} = -\frac{3}{10}\sqrt{10}$; $\csc\theta = -\frac{1}{3}\sqrt{10}$

$\cos\theta = \frac{-2}{2\sqrt{10}} = -\frac{1}{10}\sqrt{10}$; $\sec\theta = -\sqrt{10}$

$\tan\theta = \frac{-6}{-2} = 3$; $\cot\theta = \frac{1}{3}$





5. $\sin\theta = -\frac{1}{6}$ $\csc\theta = -6$

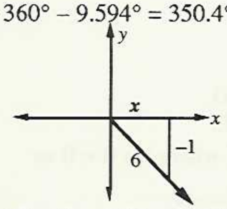
$x = \sqrt{6^2 - (-1)^2} = \sqrt{35}$

$\cos\theta = \frac{x}{6} = \frac{\sqrt{35}}{6}$ $\sec\theta = \frac{6}{35}\sqrt{35}$

$\tan\theta = \frac{-1}{\sqrt{35}} = -\frac{1}{35}\sqrt{35}$ $\cot\theta = -\sqrt{35}$

$\theta' = \sin^{-1}\frac{1}{6} \approx 9.594°$

$\theta = 360° - \theta' \approx 360° - 9.594° = 350.4°$



7. $246.2°$ in qIII so $\theta' = \theta - 180° = 246.2° - 180° = 66.2°$
9. $\sin 116.4° \approx 0.8957$
11. $\csc 115°20' = \dfrac{1}{\sin 115°20'} \approx 1.1064$

$115\ \boxed{+}\ 20\ \boxed{\div}\ 60\ \boxed{=}\ \boxed{\sin}\ \boxed{1/x}$

TI–81: $\boxed{(}\ \boxed{\sin}\ \boxed{(}\ 115\ \boxed{+}\ 20\ \boxed{\div}\ 60\ \boxed{)}\ \boxed{)}\ \boxed{x^{-1}}$

13. $\sec\theta = -2.0642$ so $\cos\theta = \dfrac{-1}{2.0642}$

$\theta' = \cos^{-1}\frac{1}{2.0642} \approx 61.0°$; $\cos\theta < 0$, $\sin\theta > 0$ so θ in qII;

$\theta = 180° - \theta' \approx 180° - 61.0° = 119.0°$

15. $x = r\cos\theta = 22.6\cos 261.42° \approx -3.4$ cm

$y = r\sin\theta = 22.6\sin 261.42° \approx -22.3$ cm

17. $\frac{415°}{180°} = \frac{s}{\pi}$; $s = \frac{83}{36}\pi \approx 7.24$ radians
19. $L = rs$; $L = 5 \cdot \frac{8.2}{2} = 20.5$ inches
21. $\csc 2.5 = \dfrac{1}{\sin 2.5} \approx 1.6709$
23. $A = \frac{1}{2}sr^2 = \frac{1}{2}(\frac{\pi}{3})(16^2) \approx 134.04\text{mm}^2$
25. a. $f(-4) = (-4)^2 - 3(-4) + 5 = 33$

b. $f(\sqrt{2}) = (\sqrt{2})^2 - 3\sqrt{2} + 5 = 7 - 3\sqrt{2}$

27. $\theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$; θ in qIII

$\theta = \pi + \theta' \approx \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

29. $2\tan 3\theta = 4.2$

$\tan 3\theta = 2.1$

$3\theta = \tan^{-1}2.1$

$\theta = \frac{1}{3}\tan^{-1}2.1 \approx 0.38$

31. $d = \frac{1}{3}\cos 4(\frac{\pi}{16}) - \frac{1}{4}\sin 4(\frac{\pi}{16})$

$= \frac{1}{3}\cos\frac{\pi}{4} - \frac{1}{4}\sin\frac{\pi}{4}$

$= \frac{1}{3} \cdot \frac{\sqrt{2}}{2} - \frac{1}{4} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{6} - \frac{\sqrt{2}}{8} = \frac{4\sqrt{2}}{24} - \frac{3\sqrt{2}}{24}$

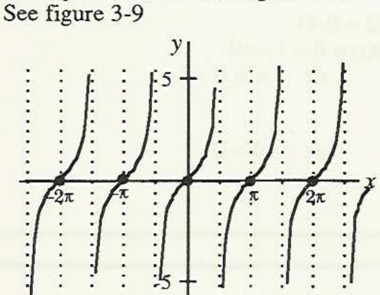
$= \frac{\sqrt{2}}{24} \approx 0.0589$

Chapter 3

Exercise 3-1

1. a. See figure 3-3; b. See figure 3-6;
   c. See figure 3-9

5. $k\pi$



9. $-\frac{5\pi}{3}$ $\sin(-\frac{5\pi}{3}) = -\sin\frac{5\pi}{3} = -(-\frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{2}$

$\cos(-\frac{5\pi}{3}) = \cos\frac{5\pi}{3} = \frac{1}{2}$

$\tan(-\frac{5\pi}{3}) = -\tan\frac{5\pi}{3} = -(-\sqrt{3}) = \sqrt{3}$

In problems 10–25 remember that $(-x)^n$ is the same as x^n if n is even and $-x^n$ if n is odd. Also a function cannot be both even and odd. (The only exception is the function $f(x) = 0$.)
Remember that $\sin(-x) = -\sin x$, $\cos(-x) = \cos x$, $\tan(-x) = -\tan x$.

13. $f(x) = -x^2$

$f(-x) = -3(-x)^2 = -3x^2$

$f(-x) = f(x)$ so f is even.

17. $f(x) = 3x - 2x^3$

$f(-x) = 3(-x) - 2(-x)^3 = -3x - (-2x^3) = -3x + 2x^3$

$-f(x) = -(3x - 2x^3) = -3x + 2x^3$

$f(-x) = -f(x)$ so f is odd

21. $f(x) = \dfrac{x^5 - x^3}{x}$

$f(-x) = \dfrac{(-x)^5 - (-x)^3}{-x} = \dfrac{-x^5 - (-x^3)}{-x}$

$= \dfrac{-x^5 + x^3}{-x} = \dfrac{-(x^5 - x^3)}{-x} = \dfrac{x^5 - x^3}{x}$

$f(-x) = f(x)$ so f is even.

25. $f(x) = \dfrac{\sin x}{x}$

$f(-x) = \dfrac{\sin(-x)}{-x} = \dfrac{-\sin x}{-x} = \dfrac{\sin x}{x}$

$f(-x) = f(x)$ so f is even.

Exercise 3–2

1. See figures 3.11, 3.12, 3.14.
3. $\csc(-x) = \dfrac{1}{\sin(-x)} = \dfrac{1}{-\sin(x)} = -\dfrac{1}{\sin x} = -\csc x$
5. $\cot(-x) = \dfrac{1}{\tan(-x)} = \dfrac{1}{-\tan(x)} = -\dfrac{1}{\tan x} = -\cot x$

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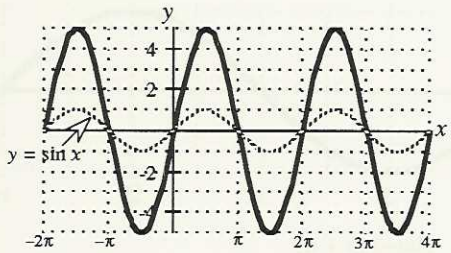
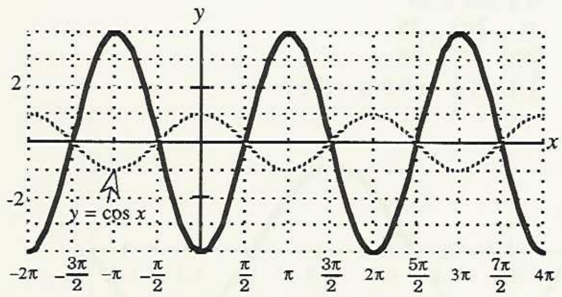
Exercise 3–3

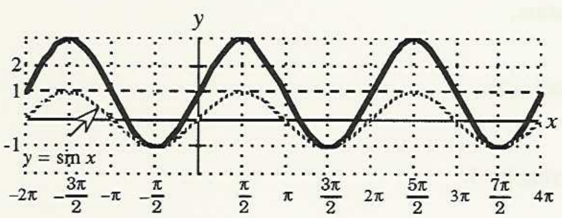
1. $y = 5\sin x$ Amplitude is 5.



5. $y = -4\cos x$ Amplitude is 4.
Graph is flipped about the x–axis relative to the graph of $y = \cos x$.



9. $y = 2\sin x + 1$ Amplitude is 2.
Graph is raised vertically 1 unit.



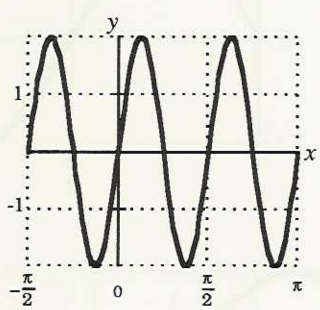
13. $y = 2\sin 4x$ Amplitude is 2.

$0 \le 4x \le 2\pi$

$\frac{0}{4} \le \frac{4x}{4} \le \frac{2\pi}{4}$ Divide each member by 4.

$0 \le x \le \frac{\pi}{2}$; one basic sine cycle between 0 and $\frac{\pi}{2}$.

Phase shift is 0; period is $\frac{\pi}{2}$.



17. $y = \frac{2}{3}\sin(3x + \pi)$ Amplitude is $\frac{2}{3}$.

$0 \le 3x + \pi \le 2\pi$

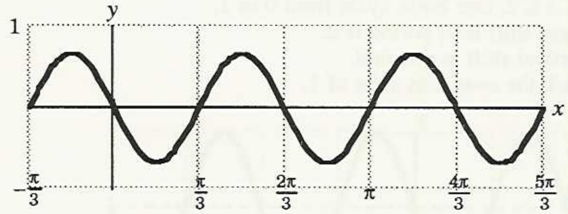
$-\pi \le 3x \le \pi$ Subtract π from each member.

$-\frac{\pi}{3} \le x \le \frac{\pi}{3}$ Divide each member by 3.

One basic cycle between $-\frac{\pi}{3}$ and $\frac{\pi}{3}$.

Phase shift is $-\frac{\pi}{3}$; period is $\frac{\pi}{3} - (-\frac{\pi}{3}) = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$.

For convenience mark the x–axis in terms of $\frac{\pi}{3}$.



21. $y = -\cos\left(2x + \frac{\pi}{2}\right)$ Amplitude is 1.

Graph is reflected about the x–axis relative to the graph of $y = \cos x$.

$0 \le 2x + \frac{\pi}{2} \le 2\pi$

$-\frac{\pi}{2} \le 2x \le 2\pi - \frac{\pi}{2}$ Subtract $\frac{\pi}{2}$ from each member.

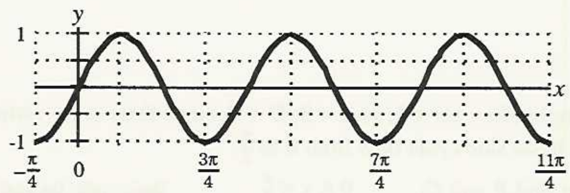
$-\frac{\pi}{2} \le 2x \le \frac{3\pi}{2}$

$\frac{1}{2}\left(-\frac{\pi}{2}\right) \le \frac{1}{2}(2x) \le \frac{1}{2} \cdot \frac{3\pi}{2}$ Multiply each member by $\frac{1}{2}$.

$-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ One basic cycle between $-\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

Phase shift is $-\frac{\pi}{4}$; period is $\frac{3\pi}{4} - (-\frac{\pi}{4}) = \pi$.

Mark the x–axis in terms of $\frac{\pi}{4}$.



25. $y = \cos 2\pi x$ Amplitude is 1.

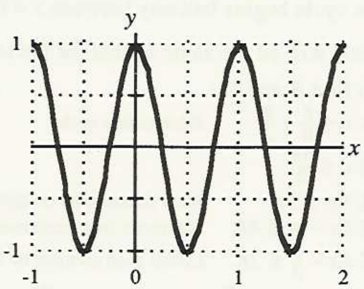
$0 \le 2\pi x \le 2\pi$

$0 \le x \le 1$ Divide each member by 2π.

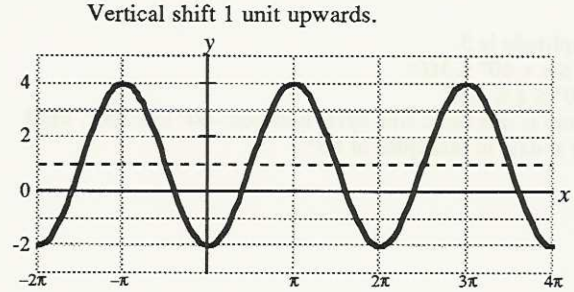
Basic cycle from 0 to 1.

Phase shift is 0; period is $1 - 0 = 1$.

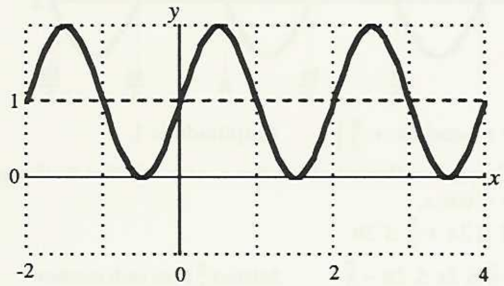
Mark the x–axis in multiples of $\frac{1}{2}$.



29. $y = -3\cos x + 1$ Amplitude is 3.
The graph is flipped about the horizontal line $y = 1$.
One basic cycle between 0 and 2π. Phase shift is 0; period is 2π.
Vertical shift 1 unit upwards.



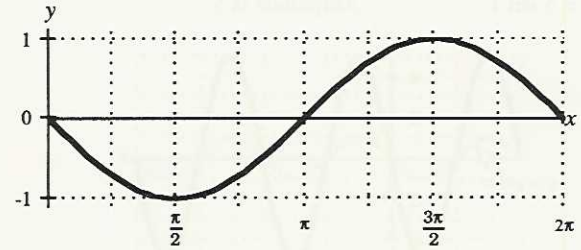
33. $y = \sin \pi x + 1$ Amplitude is 1.
$0 \le \pi x \le 2\pi$
$0 \le x \le 2$; one basic cycle from 0 to 2.
Phase shift is 0; period is 2.
Vertical shift is one unit.
Mark the x–axis in units of 1.



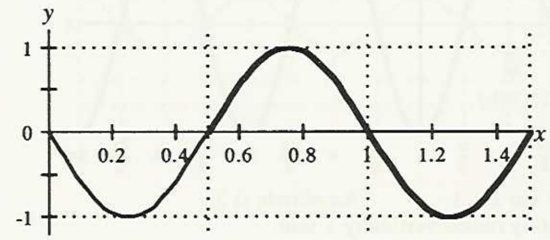
37. $y = -\cos(-3x)$
$y = -[\cos 3x]$
$y = -\cos 3x$

41. $y = \sin(-x) - 3$
$y = -\sin x - 3$

45. $y = \sin(-x)$
$y = -\sin x$



49. $y = -\sin(-2\pi x + \pi)$
$y = -\sin[-(2\pi x - \pi)]$
$y = \sin(2\pi x - \pi)$
$0 \le 2\pi x - \pi \le 2\pi$
$\pi \le 2\pi x \le 3\pi$
$\frac{\pi}{2\pi} \le \frac{2\pi x}{2\pi} \le \frac{3\pi}{2\pi}$
$\frac{1}{2} \le x \le \frac{3}{2}$, or $0.5 \le x \le 1.5$

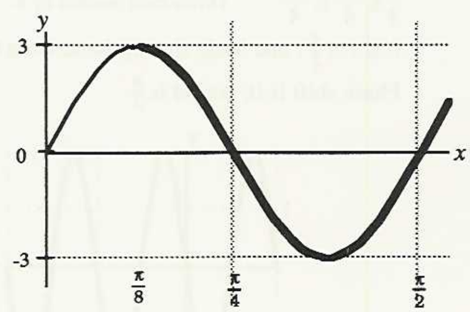


53. amplitude = $|A| = 3$, so $A = 3$; $D = 0$, since there is no vertical translation.
A basic sine cycle runs from 0 to $\frac{\pi}{2}$.

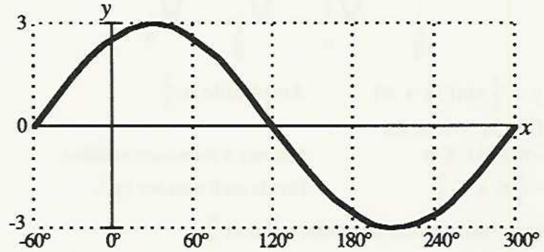
To find B and C: $0 \le x \le \frac{\pi}{2}$ Basic cycle; the goal is to convert $\frac{\pi}{2}$ to 2π.
$0 \le 2x \le \pi$ Multiply each term by 2.
$0 \le 4x \le 2\pi$ Multiply each term by 2.

Of course we could have combined the previous two steps by multiplying by 4.
Thus $Bx + C = 4x$, so $B = 4$, $C = 0$.
The equation is $y = 3 \sin 4x$.

57. $A = 3, D = 0$
A cosine cycle begins halfway between $x = 0$ and $x = \frac{\pi}{4}$, at $\frac{\pi}{8}$.
The period will be the same as that for the sine function, $\frac{\pi}{2}$.
Thus, to find B and C :
$\frac{\pi}{8} \le x \le \frac{\pi}{8} + \frac{\pi}{2}$ Basic cosine cycle.
$\frac{\pi}{8} \le x \le \frac{5\pi}{8}$
$\pi \le 8x \le 5\pi$ Clear fractions by multiplying each member by 8.
$0 \le 8x - \pi \le 4\pi$ Subtract π from each member so the left member is 0.
$0 \le 4x - \frac{\pi}{2} \le 2\pi$ Divide each member by 2, so the right member is 2π.
Thus $Bx + C = 4x - \frac{\pi}{2}$, so $B = 4$, $C = -\frac{\pi}{2}$.
The equation is $y = 3\cos\left(4x - \frac{\pi}{2}\right)$



61. amplitude is 3.
$0° \le x + 60° \le 360°$
$-60° \le x \le 300°$
There is one basic sine cycle between $-60°$ and $300°$. Mark the x–axis in multiples of $60°$.



65. $90° \le x \le 90° + 54°$

One cycle is between the phase shift and phase shift plus period.

$0° \le x - 90° \le 54°$

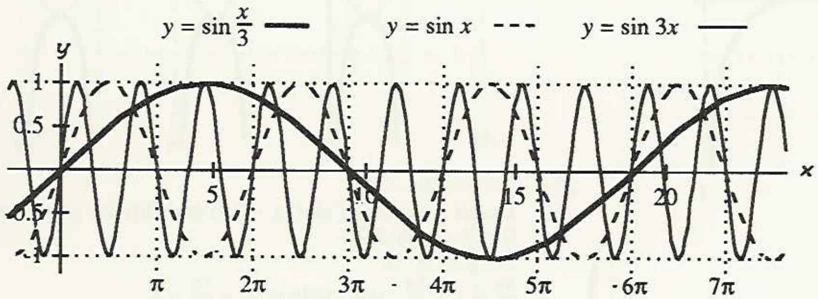
Subtract 90° so the left member is 0.

$\frac{360°}{54°} \cdot 0° \le \frac{360°}{54°} \cdot (x - 90°) \le \frac{360°}{54°} \cdot 54°$ Multiply each term by $\frac{360°}{54°}$, so the right member becomes 360°.

$0° \le \frac{20}{3}x - 600° \le 360°$

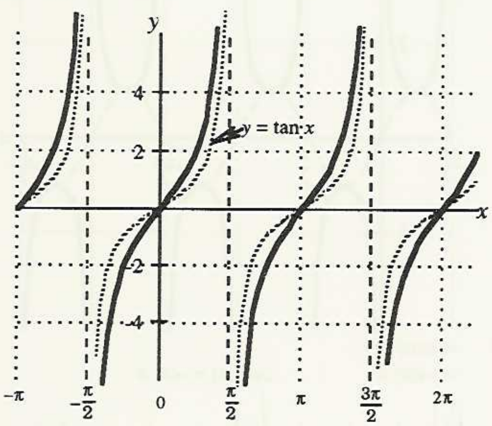
$y = 60 \sin\left(\frac{20}{3}x - 600°\right)$

69.



Exercise 3–4

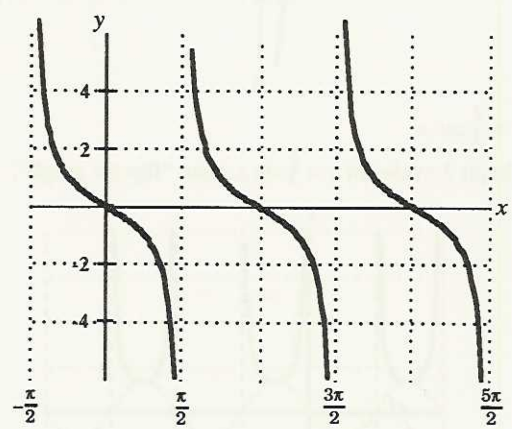
1. $y = 5 \tan x$



5. $y = \cot\left(x - \frac{\pi}{2}\right)$

$0 \le x - \frac{\pi}{2} \le \pi$

$\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ Basic cycle.

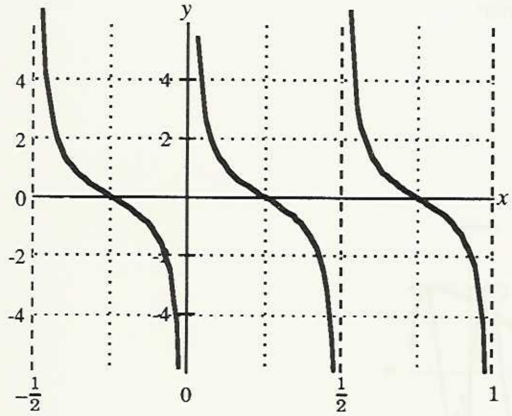


9. $y = \cot 2\pi x$

$0 \le 2\pi x \le \pi$

$0 \le x \le \frac{\pi}{2\pi}$

$0 \le x \le \frac{1}{2}$



13. $y = -\cot(-\pi x)$

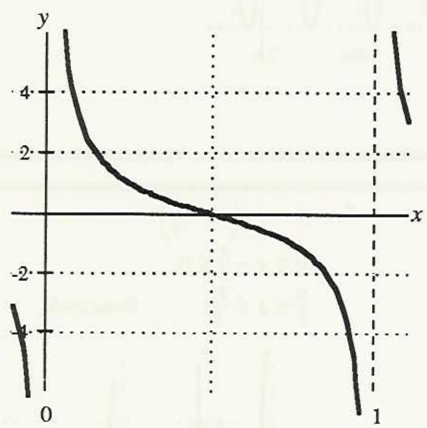
$= -[-\cot \pi x]$

$= \cot \pi x$

$0 \le \pi x \le \pi$

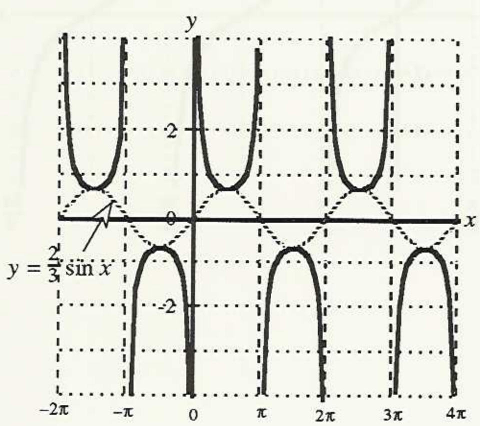
$0 \le x \le 1$

Divide each term by π.



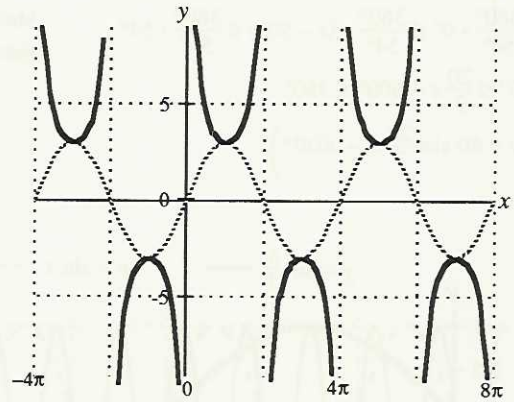
17. $y = \frac{2}{3} \csc x$

Graph 3 cycles of $y = \frac{2}{3} \sin x$, then "flip the graph".



21. $y = 3 \csc \frac{x}{2}$

Graph 3 cycles of $3 \sin \frac{x}{2}$ and "flip".



25. $y = \csc(2x - 3\pi)$

Graph 3 cycles of $\sin(2x - 3\pi)$ and flip the graph over.

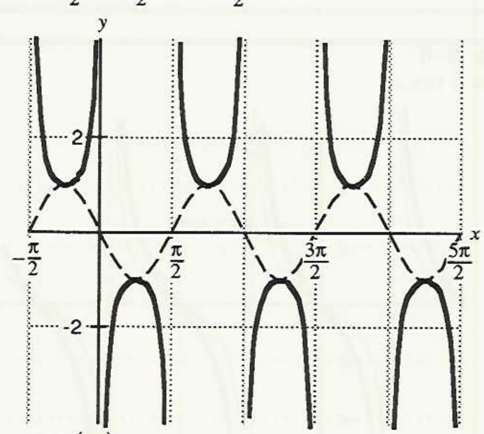
$0 \le 2x - 3\pi \le 2\pi$

$3\pi \le 2x \le 5\pi$

$\frac{3\pi}{2} \le x \le \frac{5\pi}{2}$; one cycle is $\frac{5\pi}{2} - \frac{3\pi}{2} = \pi$

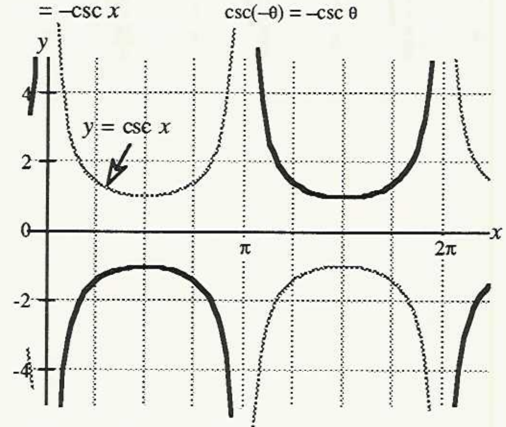
Subtracting π tells where other cycles begin.

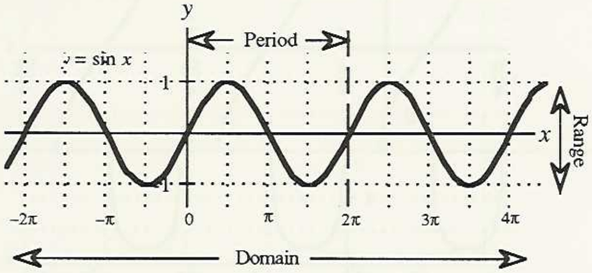
$\frac{3\pi}{2} - \pi = \frac{\pi}{2}$, and $\frac{\pi}{2} - \pi = -\frac{\pi}{2}$



29. $y = \csc(-x)$

$= -\csc x$ $\qquad \csc(-\theta) = -\csc \theta$



1. 

Domain: R; Range: $-1 \le y \le 1$; Period: 2π.

3. $\cos(-\frac{\pi}{6}) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ cosine is even so $\cos(-x) = \cos x$

5. $\sin(-\frac{5\pi}{6}) = -\sin\frac{5\pi}{6} = -\frac{1}{2}$

7. $f(x) = \dfrac{x^2 - 1}{x}$

$f(-x) = \dfrac{(-x)^2 - 1}{-x} = \dfrac{x^2 - 1}{-x} \bullet \dfrac{-1}{-1} = \dfrac{-x^2 + 1}{x}$

$-f(x) = -\dfrac{x^2 - 1}{x} = \dfrac{-x^2 + 1}{x}$

$f(-x) = -f(x)$, so f is odd.

9. $f(x) = \tan x \bullet \cos x$

$f(-x) = \tan(-x) \bullet \cos(-x)$

$= -\tan x \bullet \cos x$ $\tan(-x) = -\tan x$; $\cos(-x) = \cos x$.

$= -(\tan x \bullet \cos x)$

$= -f(x)$.

Since $f(-x) = -f(x)$, f is an odd function.

11. domain: $x \ne k\pi$, k an integer; range: R

13. $f(x) = \sec x \bullet \sin^2 x + x^4$

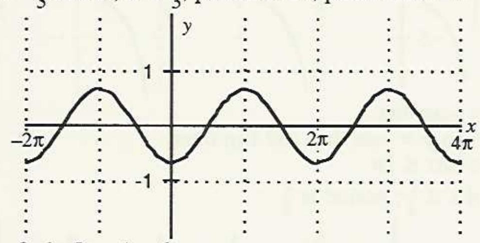
$f(-x) = \sec(-x) \bullet [\sin(-x)]^2 + (-x)^4$

$= \sec x \bullet [-\sin x]^2 + x^4$

$= \sec x \bullet \sin^2 x + x^4$

$f(-x) = f(x)$ so f is even.

15. $y = -\frac{2}{3}\cos x$; $A = \frac{2}{3}$, period $= 2\pi$, phase shift $= 0$

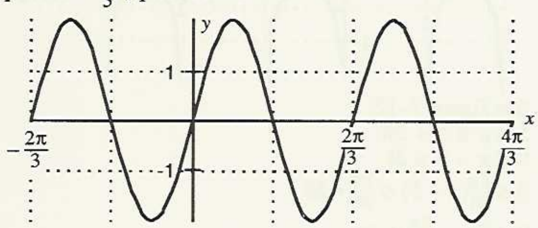


17. $y = 2 \sin 3x$; $A = 2$

$0 \le 3x \le 2\pi$

$0 \le x \le \frac{2\pi}{3}$ divide each member by 3

period $= \frac{2\pi}{3}$; phase shift $= 0$



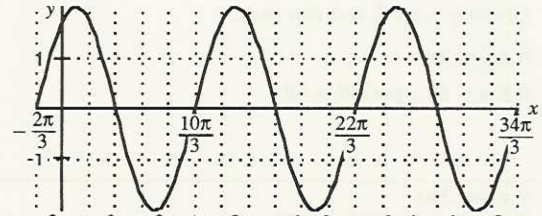
19. $y = 2\sin(\frac{x}{2} + \frac{\pi}{3})$; $A = 2$

$0 \le \frac{x}{2} + \frac{\pi}{3} \le 2\pi$

$0 \le 3x + 2\pi \le 12\pi$ Multiply each member by 3

$-2\pi \le 3x \le 10\pi$

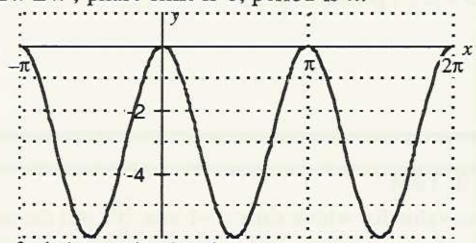
$-\frac{2\pi}{3} \le x \le \frac{10\pi}{3}$; phase shift is $-\frac{2\pi}{3}$; period is $\frac{10\pi}{3} - (-\frac{2\pi}{3}) = 4\pi$



21. $y = 3\cos 2x - 3$; $A = 3$; vertical translation is -3.

$0 \le 2x \le 2\pi$

$0 \le x \le \pi$; phase shift is 0, period is π.



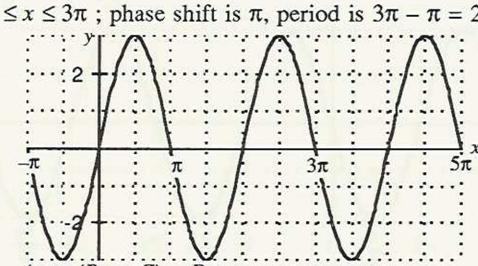
23. $y = 3\sin(-x + \pi)$; $A = 1$

$y = 3\sin[-(x - \pi)]$

$y = -3\sin(x - \pi)$

$0 \le x - \pi \le 2\pi$

$\pi \le x \le 3\pi$; phase shift is π, period is $3\pi - \pi = 2\pi$



25. $y = A\cos(Bx + C) + D$

$A = 3$, A cycle starts at $\frac{1}{2} \bullet \frac{\pi}{3} = \frac{\pi}{6}$; cycle length is $\frac{2\pi}{3} - (-\frac{\pi}{3}) = \pi$, so one basic cycle is between $\frac{\pi}{6}$ and $\frac{\pi}{6} + \pi = \frac{7\pi}{6}$. $D = -1$.

$\frac{\pi}{6} \le x \le \frac{7\pi}{6}$

$0 \le x - \frac{\pi}{6} \le \pi$ Subtract $\frac{\pi}{6}$ from each member.

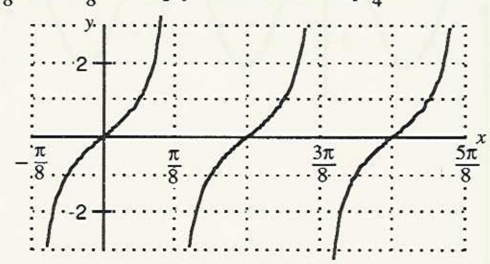
$0 \le 2x - \frac{\pi}{3} \le 2\pi$ Multiply each member by 2.

$y = 3\cos(2x - \frac{\pi}{3}) - 1$

27. $y = \tan 4x$

$-\frac{\pi}{2} \le 4x \le \frac{\pi}{2}$

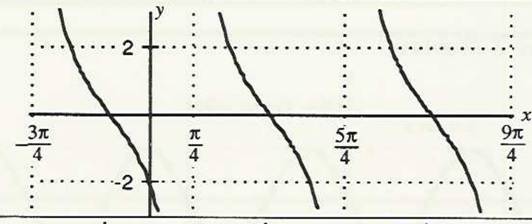
$-\frac{\pi}{8} \le x \le \frac{\pi}{8}$ multiply each member by $\frac{1}{4}$



29. $y = 2\cot(x - \frac{\pi}{4})$

$0 \le x - \frac{\pi}{4} \le \pi$

$\frac{\pi}{4} \le x \le \frac{5\pi}{4}$

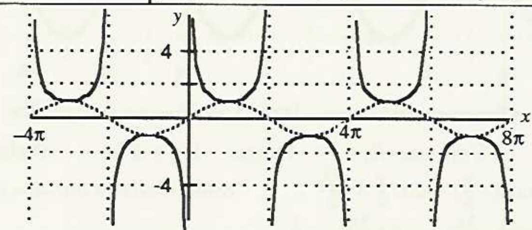


31. $y = \csc \frac{x}{2}$

Graph $y = \sin \frac{x}{2}$ and flip over.

$0 \le \frac{x}{2} \le 2\pi$

$0 \le x \le 4\pi$; period is 4π

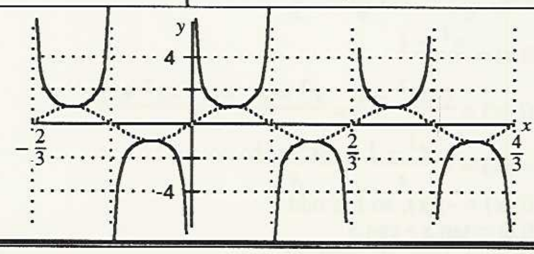


33. $y = \csc 3\pi x$

Graph $y = \sin 3\pi x$ and flip over.

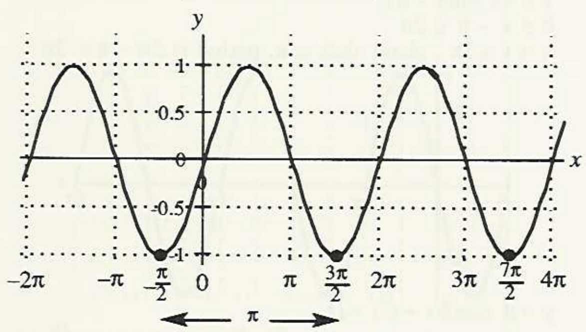
$0 \le 3\pi x \le 2\pi$

$0 \le x \le \frac{2}{3}$; period is $\frac{2}{3}$



Chapter 3 Test

1. One value for which $\sin x = -1$ is at $\frac{3\pi}{2}$. All the other values are integer multiples of 2π units from this value. Thus the values are $x = \frac{3\pi}{2} + 2k\pi$, k an integer.



3. $f(x) = x + \sin x$

$f(-x) = (-x) + \sin(-x)$

$= (-x) + (-\sin x)$

$= -(x + \sin x)$

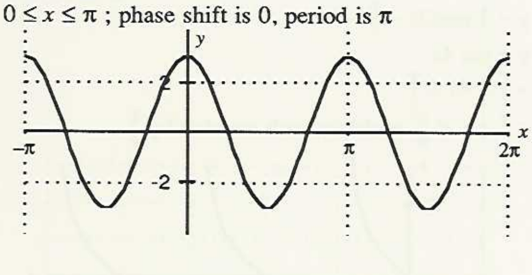
$= -f(x)$.

Since $f(-x) = -f(x)$, f is an odd function.

5. $y = 3\cos 2x$; $A = 3$

$0 \le 2x \le 2\pi$

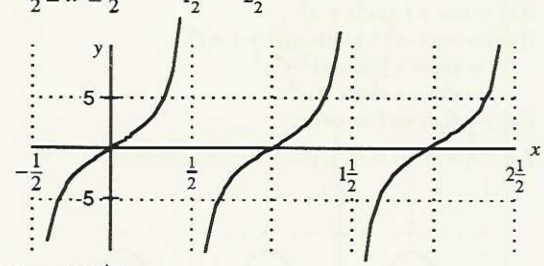
$0 \le x \le \pi$; phase shift is 0, period is π



7. $3\tan \pi x$

$-\frac{\pi}{2} \le \pi x \le \frac{\pi}{2}$

$-\frac{1}{2} \le x \le \frac{1}{2}$ $\quad 1\frac{1}{2} \quad 2\frac{1}{2}$

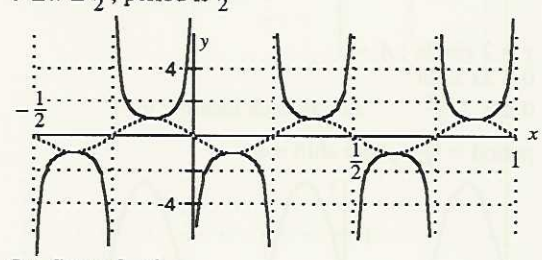


9. $y = -\csc 4\pi x$

Graph $y = -\sin 4\pi x$ and flip over.

$0 \le 4\pi x \le 2\pi$

$0 \le x \le \frac{1}{2}$; period is $\frac{1}{2}$



11. See figure 3–12.

13. $5 \le x \le 5 + 28$

$0 \le x - 5 \le 28$

$0 \le \frac{2\pi}{28}(x - 5) \le \frac{2\pi}{28} \cdot 28$

$0 \le \frac{\pi}{14}x - \frac{5\pi}{14} \le 2\pi$

$y = \sin(\frac{\pi}{14}x - \frac{5\pi}{14})$, x in days

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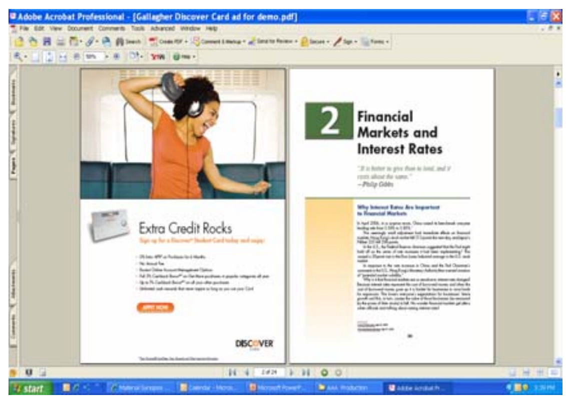

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Exersise 4–1

1. $f(x) = 2x - 7; \quad g(x) = \frac{1}{2}x + 3\frac{1}{2}$

(1) Let $y = f(x) = 2x - 7$. Show $g(y) = x$:

$$g(y) = \tfrac{1}{2}y + 3\tfrac{1}{2} = \tfrac{1}{2}(2x - 7) + \tfrac{7}{2} = x$$

(2) Let $y = g(x) = \frac{1}{2}x + \frac{7}{2}$. Show $f(y) = x$:

$$f(y) = 2y - 7 = 2(\tfrac{1}{2}x + \tfrac{7}{2}) - 7 = x$$

5. $f(x) = 2x - 5; \quad g(x) = \frac{1}{2}(x + 5)$

(1) Let $y = f(x) = 2x - 5$. Show $g(y) = x$:

$$g(y) = \tfrac{1}{2}(y + 5) = \tfrac{1}{2}[(2x - 5) + 5] = x$$

(2) Let $y = g(x) = \frac{1}{2}(x + 5)$. Show $f(y) = x$:

$$f(y) = 2y - 5 = 2[\tfrac{1}{2}(x + 5)] - 5 = x$$

9. $f(x) = 7 - \frac{3}{x}; \quad g(x) = \frac{3}{7 - x}$

(1) Let $y = f(x) = 7 - \frac{3}{x}$. Show $g(y) = x$:

$$g(y) = \frac{3}{7 - y} = \frac{3}{7 - \left(7 - \frac{3}{x}\right)} = \frac{3}{\frac{3}{x}} = x$$

(2) Let $y = g(x) = \frac{3}{7 - x}$. Show $f(y) = x$:

$$f(y) = 7 - \frac{3}{y} = 7 - \frac{3}{\frac{3}{7 - x}} = 7 - (7 - x) = x$$

13. $f(x) = x^2 - 9, x \ge 0; \quad g(x) = \sqrt{x + 9}$

(1) Let $y = f(x) = x^2 - 9$. Show $g(y) = x$:

$$g(y) = \sqrt{y + 9} = \sqrt{(x^2 - 9) + 9} = \sqrt{x^2} = x \text{ if } x \ge 0.$$

(2) Let $y = g(x) = \sqrt{x + 9}$. Show $f(y) = x$:

$$f(y) = y^2 - 9 = (\sqrt{x + 9})^2 - 9 = (x + 9) - 9 = x$$

17. $f(x) = x^2 - 2x + 3, x \ge 1; \quad g(x) = \sqrt{x - 2} + 1$

(1) Let $y = f(x) = x^2 - 2x - 3$. Show $g(y) = x$:

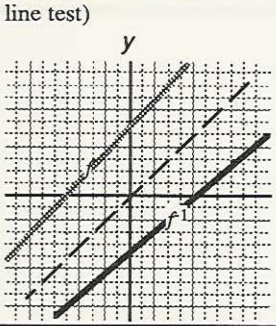
$$g(y) = \sqrt{y - 2} + 1 = \sqrt{(x^2 - 2x + 3) - 2} + 1 = \sqrt{x^2 - 2x + 1} + 1 = \sqrt{(x - 1)^2} + 1 = x - 1 + 1 = x.$$

(2) Let $y = g(x) = \sqrt{x - 2} + 1$. Show $f(y) = x$:

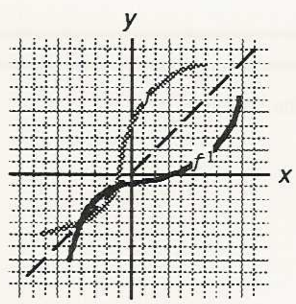
$$f(y) = y^2 - 2y + 3 = (\sqrt{x-2} + 1)^2 - 2(\sqrt{x-2} + 1) + 3 = [(x - 2) + 2\sqrt{x-2} + 1] - 2\sqrt{x-1} - 2 + 3 = x$$

21. Function, not one to one (passes the vertical line test, fails the horizontal line test)

25.



29.

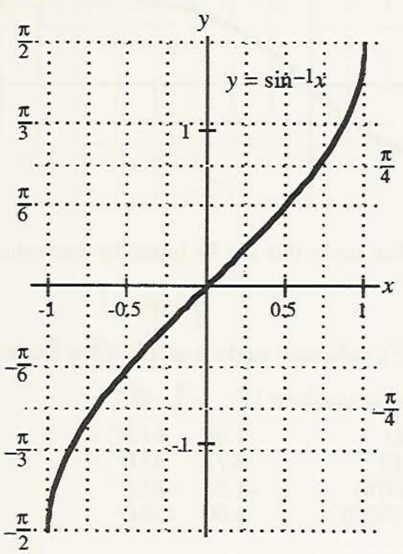


33. $A(x) = 4(x + 4)$

$$y = 4(x + 4)$$
$$x = 4(y + 4)$$
$$\frac{x}{4} = y + 4$$
$$y = \frac{x}{4} - 4$$
$$A^{-1}(x) = \frac{x}{4} - 4$$

Exercise 4–2

1.



5. $\sin^{-1}0 = 0$ since $\sin 0 = 0$ and $-\frac{\pi}{2} \le 0 \le \frac{\pi}{2}$; 0 or 0°

Use degree mode to obtain answers in degrees and radian mode to obtain answers in radians.

9. $\sin^{-1}0.8823$ 1.08   61.9°

13. $\sin^{-1}(-0.9976)$ −1.50   −86.0°

.9976 $\boxed{+/-}$ $\boxed{\text{2nd}}$ $\boxed{\sin^{-1}}$

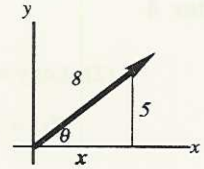
TI–81: $\boxed{\text{2nd}}$ $\boxed{\sin^{-1}}$ $\boxed{(}$ $\boxed{(-)}$ .9976 $\boxed{)}$ $\boxed{\text{ENTER}}$

17. $\tan(\arcsin \frac{5}{8})$

$x = \sqrt{8^2 - 5^2} = \sqrt{39}$; $\theta = \arcsin \frac{5}{8}$;

$\tan \theta = \frac{5}{x} = \frac{5}{\sqrt{39}} \cdot \frac{\sqrt{39}}{\sqrt{39}}$

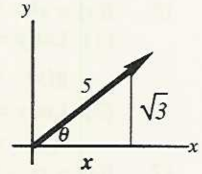
$= \frac{5\sqrt{39}}{39}$ or $\frac{5}{39}\sqrt{39}$



21. $\cot\left(\sin^{-1} \frac{\sqrt{3}}{5}\right)$

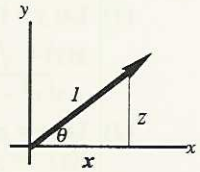
$x = \sqrt{22}$; $\tan \theta = \frac{\sqrt{3}}{x} = \frac{\sqrt{3}}{\sqrt{22}}$;

$\cot \theta = \frac{1}{\tan \theta} = \frac{\sqrt{22}}{\sqrt{3}} = \frac{\sqrt{66}}{3}$.



25. $\cos(\sin^{-1} z), z > 0$

$x = \sqrt{1 - z^2}$; $\cos \theta = \frac{x}{1} = \sqrt{1 - z^2}$.



29. $\sec(\arcsin \sqrt{2z})$

Note that $\sqrt{2z} \geq 0$, so

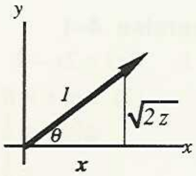
$\theta = \arcsin \sqrt{2z}$ terminates in quadrant I (or is quadrantal).

$x = \sqrt{1^2 - (\sqrt{2z})^2} = \sqrt{1 - 2z}$.

$\cos \theta = \frac{x}{1} = x = \sqrt{1 - 2z}$;

$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\sqrt{1 - 2z}} \cdot \frac{\sqrt{1 - 2z}}{\sqrt{1 - 2z}} = \frac{\sqrt{1 - 2z}}{1 - 2z}$

Note: If $2z = 0$ the angle is quadrantal, but the result is still valid.



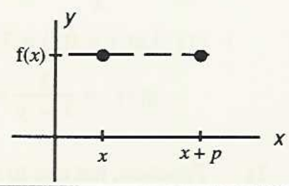
33. $\arcsin\left(\cos \frac{2\pi}{3}\right)$

$\arcsin(-\frac{1}{2})$

$-\arcsin \frac{1}{2}$

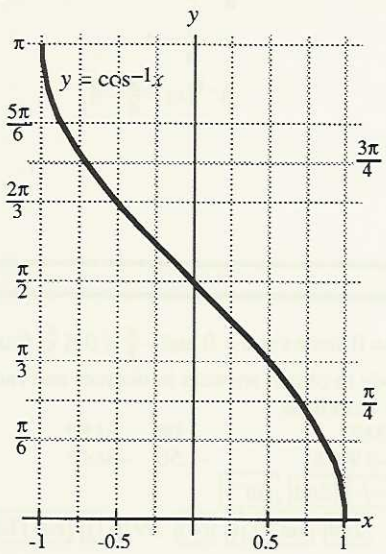
$-\frac{\pi}{6}$

37. A periodic function cannot be one to one. There are two ways to see this. First of all, let x be in the domain of f. Then there is a number $p > 0$ such that $f(x + kp) = f(x)$. This means there are two ordered pairs in f where the second element is repeated: $(x, f(x)$ and $(x + kp, f(x))$. The second way to see this is pictorially. The graph of f must fail the horizontal lines test.

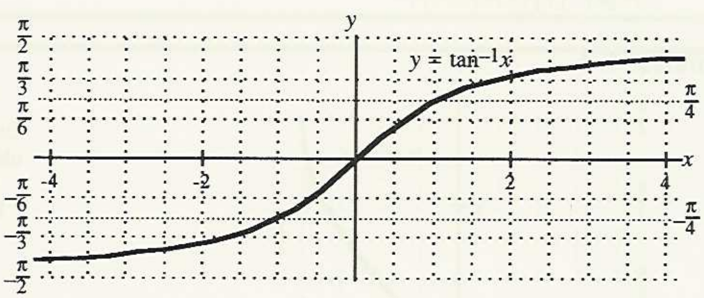


Exercise 4–3

1. See problem 1, exercise 4–2 for the inverse sine function.



b.



c.

The following table is taken from the text. It will be used in the following problems.

θ	0, 0°	$\frac{\pi}{6}$, 30°	$\frac{\pi}{4}$, 45°	$\frac{\pi}{3}$, 60°	$\frac{\pi}{2}$, 90°
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Not defined

Note: A calculator will give exact answers in degree mode when the reference angle is 30°, 45° or 60°. If you know the corresponding radian mode this can be found by converting degrees to radians.

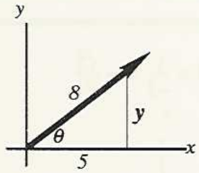
5. $\cos^{-1} 0$ $\quad \frac{\pi}{2}$, 90°

9. $\arctan(-\sqrt{3})$; reference angle is $\arctan \sqrt{3} = \frac{\pi}{3}$ or 60° but the angle is in quadrant IV: $-\frac{\pi}{3}$, –60°

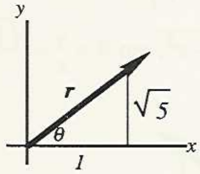
13.	$\sin^{-1}0.8823$	1.08	61.9°
17.	$\tan^{-1} 0.9316$	0.75	43.0°
21.	$\sin^{-1}(-0.9976)$	–1.50	–86.0°
25.	$\arccos(-0.9902)$	3.00	172.0°

29. $\sin(\arccos \frac{5}{8})$

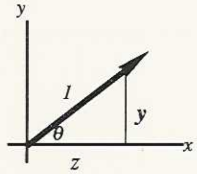
$y = \sqrt{39}$; $\sin \theta = \frac{y}{8} = \frac{\sqrt{39}}{8}$



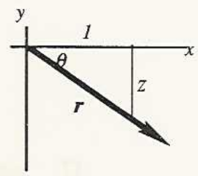
33. $\sin(\tan^{-1}\sqrt{5})$
$r = \sqrt{6}$; $\sin\theta = \frac{\sqrt{5}}{r} = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{30}}{6}$.



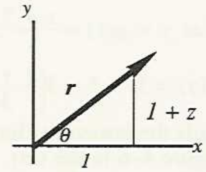
37. $\sin(\cos^{-1} z), z > 0$
$y = \sqrt{1 - z^2}$; $\sin\theta = \frac{y}{1} = y$
$= \sqrt{1 - z^2}$.



41. $\cos(\arctan z), z < 0$
$r = \sqrt{1 + z^2}$; $\cos\theta = \frac{1}{r} = \frac{1}{\sqrt{1 + z^2}}$

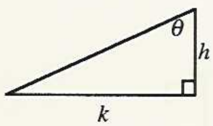


45. $\sec[\tan^{-1}(1 + z)], z > 0$
$r = \sqrt{1^2 + (1 + z)^2} =$
$\sqrt{z^2 + 2z + 2}$.
$\cos\theta = \frac{1}{r}$; $\sec\theta = \frac{1}{\cos\theta}$
$= r = \sqrt{z^2 + 2z + 2}$.



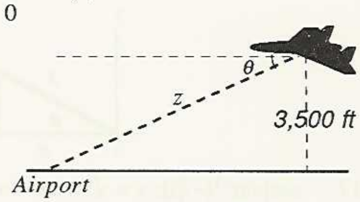
49. $\tan^{-1}(\sin\frac{\pi}{2})$
$\tan^{-1} 1$
$\frac{\pi}{4}$

53. $\arccos(\tan\frac{5\pi}{4})$
$\arccos(1)$
0

57. 

$\tan\theta = \frac{k}{h}$, so $\theta = \tan^{-1}\frac{k}{h}$.

61. $\sin\theta = \frac{3500}{z}$, so
$\theta = \sin^{-1}\frac{3500}{z}$.



65. $\tan\theta = 4.1$
$\theta = \tan^{-1} 4.1$

69. $\frac{\tan\theta}{5} = 10$
$\tan\theta = 50$
$\theta = \tan^{-1} 50$

73. $\sin\frac{3\theta}{2} = -0.56$
$\frac{3\theta}{2} = \sin^{-1}(-0.56)$
$\theta = \frac{2}{3}\sin^{-1}(-0.56)$

77. $\frac{6\sin 5\theta}{5} = \frac{10}{13}$
$6\sin 5\theta = \frac{50}{13}$
$\sin 5\theta = \frac{1}{6}\cdot\frac{50}{13}$
$\sin 5\theta = \frac{25}{39}$
$5\theta = \sin^{-1}\frac{25}{39}$
$\theta = \frac{1}{5}\sin^{-1}\frac{25}{39}$

81. $\sin(2x + 3) = 0.6$
$2x + 3 = \sin^{-1} 0.6$
$2x = \sin^{-1} 0.6 - 3$
$x = \frac{1}{2}(\sin^{-1} 0.6 - 3)$

Exercise 4–4

1. $\csc^{-1}2 = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ or 30°

5. $\sec^{-1}(-2) = \cos^{-1}(-\frac{1}{2}) = \pi - \cos^{-1}\frac{1}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ or 120°

9. $\operatorname{arccot} 0 = \frac{\pi}{2} - \arctan 0 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$ or 90°

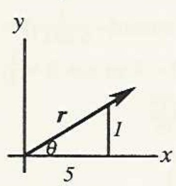
In the following problems use degree mode to obtain degree results and radian mode to obtain radian results.

13. $\operatorname{arcsec}(-2.9986) = \cos^{-1}(-\frac{1}{2.9986}) \approx 1.91, 109.5°$
2.9986 $\boxed{1/x}$ $\boxed{+/-}$ $\boxed{2nd}$ $\boxed{\cos^{-1}}$
TI–81: $\boxed{2nd}$ $\boxed{\cos^{-1}}$ $\boxed{(-)}$ 2.9986 $\boxed{x^{-1}}$ $\boxed{ENTER}$

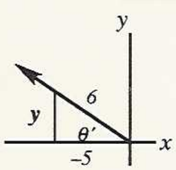
17. $\sec^{-1}(-11.1261) = \cos^{-1}(-\frac{1}{11.1261}) \approx 1.66, 95.2°$

21. $\sin(\csc^{-1} 3) = \sin(\sin^{-1}\frac{1}{3}) = \frac{1}{3}$

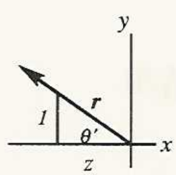
25. $\csc(\operatorname{arccot} 5)$ If $\theta = \operatorname{arccot} 5$, then $\cot\theta = 5$, and $\tan\theta = \frac{1}{5}$.
$r = \sqrt{26}$;
$\csc\theta = \frac{1}{\sin\theta}$
$= \frac{1}{\frac{1}{r}} =$
$r = \sqrt{26}$



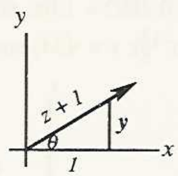
29. $\tan(\sec^{-1}(-\frac{6}{5})) = \tan(\cos^{-1}(-\frac{5}{6}))$
$y = \sqrt{11}$; $\tan\theta = \frac{y}{-5} = -\frac{\sqrt{11}}{5}$.



33. $\sec(\cot^{-1}z), z < 0$
If $\theta = \cot^{-1}z, z < 0$, then θ is an angle terminating in quadrant II, and
$\tan\theta = \frac{1}{z}$. $r = \sqrt{z^2 + 1}$;
$\sec\theta = \frac{1}{\cos\theta} =$
$\frac{1}{\frac{z}{r}} = \frac{r}{z}$
$= \frac{\sqrt{z^2 + 1}}{z}$.



37. $\tan[\sec^{-1}(z + 1)], z + 1 > 0$
$= \tan(\cos^{-1}\frac{1}{z + 1})$
$y = \sqrt{(z + 1)^2 - 1^2} = \sqrt{z^2 + 2z}$;
$\tan\theta = \frac{y}{1} = y = \sqrt{z^2 + 2z}$.



1. $f(x) = 3x - 5$; $g(x) = \dfrac{x+5}{3}$

   (1) Let $y = f(x) = 3x - 5$; show that $g(y) = x$.

   $g(y) = \dfrac{y+5}{3} = \dfrac{(3x-5)+5}{3} = x$

   (2) Let $y = g(x) = \dfrac{x+5}{3}$; show that $f(y) = x$.

   $f(y) = 3y - 5 = 3\left(\dfrac{x+5}{3}\right) - 5 = x$

3. No; fails the horizontal line test.

5. See figure 4–6 in the text.

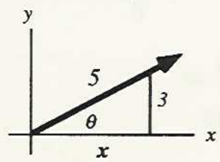
7. $\sin^{-1}(-\frac{1}{2}) = -\sin^{-1}\frac{1}{2} = -\frac{\pi}{6}, -30°$

9. $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}, 30°$

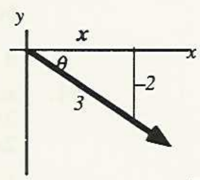
11. $\arcsin 0 = 0, 0°$

13. $\sin^{-1}0.9737 \approx 1.34, 76.8°$

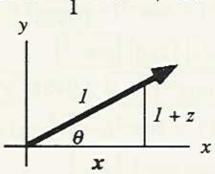
15. $\cos(\sin^{-1}\frac{3}{5})$; $x = 4$; $\cos\theta = \frac{x}{5} = \frac{4}{5}$



17. $\sec[\sin^{-1}(-\frac{2}{3})]$; $x = \sqrt{5}$; $\cos\theta = \frac{\sqrt{5}}{3}$, $\sec\theta = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$



19. $\cos(\arcsin(1+z))$, $1 + z > 0$; $x = \sqrt{1^2 - (1+z)^2}$
   $= \sqrt{-2z - z^2}$; $\cos\theta = \frac{x}{1} = x = \sqrt{-2z - z^2}$



21. domain: R ; range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

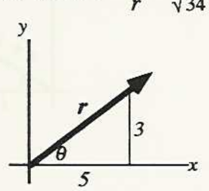
23. $\arccos\frac{\sqrt{3}}{2} = 30°, \frac{\pi}{6}$ (Remember that a calculator will give the exact answer when in degree mode.)

25. $\arcsin(-\frac{\sqrt{2}}{2}) = -\arcsin\frac{\sqrt{2}}{2} = -45°, -\frac{\pi}{4}$

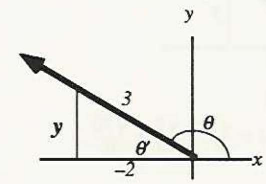
27. $\tan^{-1}(-1) = -45°, -\frac{\pi}{2}$

29. $\arccos 0.4882 \approx 1.06, 60.8°$

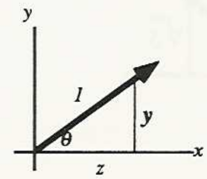
31. $\cos(\tan^{-1}\frac{3}{5})$; $r = \sqrt{34}$; $\cos\theta = \frac{5}{r} = \frac{5}{\sqrt{34}} = \frac{5}{34}\sqrt{34}$



33. $\tan[\cos^{-1}(-\frac{2}{3})]$; $y = \sqrt{5}$; $\tan\theta = \frac{y}{-2} = -\frac{\sqrt{5}}{2}$

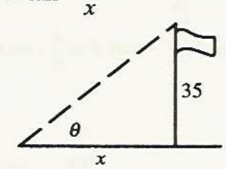


35. $\tan(\cos^{-1}z)$, $z > 0$; $y = \sqrt{1 - z^2}$; $\tan\theta = \frac{y}{z} = \frac{\sqrt{1-z^2}}{z}$



37. $\sin\theta = \frac{j}{k}$ so $\theta = \sin^{-1}\frac{j}{k}$

39. $\tan\theta = \frac{35}{x}$ so $\theta = \tan^{-1}\frac{35}{x}$



41. $\sin\theta = -0.88$
   $\theta = \sin^{-1}(-0.88)$

43. $2\tan\theta = 10$
   $\tan\theta = 5$
   $\theta = \tan^{-1}5$

45. $2\cos 3\theta = 1.4$
   $\cos 3\theta = 0.7$
   $3\theta = \cos^{-1}0.7$
   $\theta = \frac{1}{3}\cos^{-1}0.7$

47. $\sin 2\theta + 3 = 3.6$
   $\sin 2\theta = 0.6$
   $2\theta = \sin^{-1}0.6$
   $\theta = \frac{1}{2}\sin^{-1}0.6$

49. $\operatorname{arccsc}(-\frac{2\sqrt{3}}{3}) = \arcsin(-\frac{3}{2\sqrt{3}}) = \arcsin(-\frac{\sqrt{3}}{2}) = -60°, -\frac{\pi}{3}$

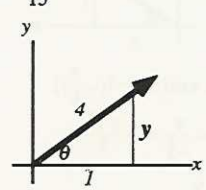
51. $\cot^{-1}(-\sqrt{3}) = \frac{\pi}{2} - \tan^{-1}(-\sqrt{3}) = \frac{\pi}{2} - (-\frac{\pi}{3}) = \frac{5\pi}{6}, 150°$

53. $\cot^{-1}1.5601 = \frac{\pi}{2} - \tan^{-1}1.5601 \approx 0.57, 32.7°$

55. $\operatorname{arccsc}(-6.6917) = \arcsin(-\frac{1}{6.6917}) \approx -0.15, -8.6°$

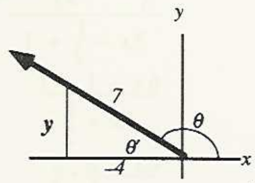
57. $\cot(\operatorname{arcsec} 4)$; $\sec\theta = 4$ so $\cos\theta = \frac{1}{4}$; $y = \sqrt{15}$

   $\cot\theta = \frac{1}{y} = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$



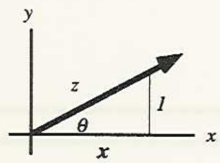
59. $\csc[\operatorname{arcsec}(-\frac{7}{4})]$; $\sec\theta = -\frac{7}{4}$ so $\cos\theta = -\frac{4}{7}$; $y = \sqrt{33}$

$\csc\theta = \frac{7}{y} = \frac{7}{\sqrt{33}} = \frac{7\sqrt{33}}{33}$



61. $\cot(\csc^{-1}z)$, $z > 0$; $\csc\theta = z$ so $\sin\theta = \frac{1}{z}$; $x = \sqrt{z^2 - 1}$

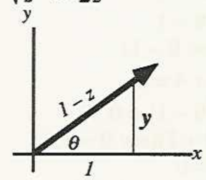
$\cot\theta = \frac{x}{1} = x = \sqrt{z^2 - 1}$



63. $\csc[\sec^{-1}(1 - z)]$, $1 - z > 0$; $\sec\theta = 1 - z$ so $\cos\theta = \frac{1}{1 - z}$;

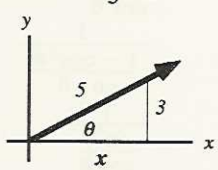
$y = \sqrt{(1 - z)^2 - 1} = \sqrt{z^2 - 2z}$; $\sin\theta = \frac{y}{1 - z}$ so

$\csc\theta = \frac{1 - z}{y} = \frac{1 - z}{\sqrt{z^2 - 2z}}$



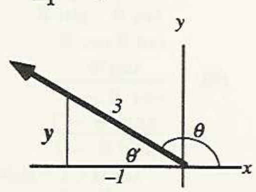
Chapter 4 test

1. See figure 4-10 in the text.
3. $\sec^{-1}(-2) = \cos^{-1}(-\frac{1}{2}) = 120°, \frac{2\pi}{3}$
5. $\csc^{-1}2 = \sin^{-1}\frac{1}{2} = 30°, \frac{\pi}{6}$
7. $\tan^{-1}1.4617 \approx 0.97, 55.6°$
9. $\operatorname{arcsec}(-1.4830) = \arccos(\frac{-1}{1.4830}) \approx 2.31, 132.4°$
11. $\cot(\sin^{-1}\frac{3}{5})$; $x = 4$; $\cot\theta = \frac{x}{3} = \frac{4}{3}$



13. $\tan[\sec^{-1}(-3)]$; $\sec\theta = -3$ so $\cos\theta = -\frac{1}{3}$

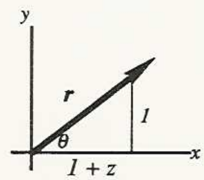
$y = 2\sqrt{2}$; $\tan\theta = \frac{y}{-1} = -y = -2\sqrt{2}$



15. $\sin[\operatorname{arccot}(1 + z)]$, $1 + z > 0$; $\cot\theta = 1 + z$ so $\tan\theta = \frac{1}{1 + z}$.

$r = \sqrt{1^2 + (1 + z)^2} = \sqrt{z^2 + 2z + 2}$; $\sin\theta = \frac{1}{r}$

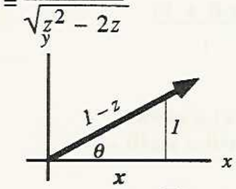
$= \frac{1}{\sqrt{z^2 + 2z + 2}}$



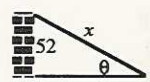
17. $\sec[\csc^{-1}(1 - z)]$, $1 - z > 0$; $\csc\theta = 1 - z$, so $\sin\theta = \frac{1}{1 - z}$;

$x = \sqrt{(1 - z)^2 - 1} = \sqrt{z^2 - 2z}$; $\cos\theta = \frac{x}{1 - z} = \frac{\sqrt{z^2 - 2z}}{1 - z}$;

$\sec\theta = \frac{1}{\cos\theta} = \frac{1 - z}{\sqrt{z^2 - 2z}}$



19. $\sin\theta = \frac{52}{x}$; $\theta = \sin^{-1}\frac{52}{x}$



21. $2\sin\theta = -1.88$

$\sin\theta = \frac{-1.88}{2} = -0.94$

$\theta = \sin^{-1}(-0.94)$

23. $4\cos(2\theta - 3) = 2.4$

$\cos(2\theta - 3) = 0.6$

$2\theta - 3 = \cos^{-1}0.6$

$2\theta = \cos^{-1}0.6 + 3$

$\theta = \frac{1}{2}(\cos^{-1}0.6 + 3)$

Chapter 5

Exercise 5-0

1. $\sin^2\theta - \sin\theta$
$\sin\theta(\sin\theta - 1)$

5. $\cos^2 x + \cos x - 20$
$(\cos x - 4)(\cos x + 5)$

9. $6\csc^2\theta - 5\csc\theta + 1$
$(2\csc\theta - 1)(3\csc\theta - 1)$

13. $\sec^4\theta - 5\sec^2\theta + 4 = 0$
$(\sec^2\theta - 4)(\sec^2\theta - 1) = 0$
$(\sec\theta - 2)(\sec\theta + 2)(\sec\theta - 1)(\sec\theta + 1) = 0$
$\sec\theta - 2 = 0$ or $\sec\theta + 2 = 0$ or $\sec\theta - 1 = 0$ or $\sec\theta + 1 = 0$
$\sec\theta = \pm 2$ or ± 1

17. $2\sec^2\theta + 3\sec\theta - 7 = 0$
$\sec\theta = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)}$
$\sec\theta = \dfrac{-3 \pm \sqrt{65}}{4}$

21. $\dfrac{2\left(3x - \frac{1}{2}\right)^2 - 3\left(3x - \frac{1}{2}\right) + 1}{\left(3x - \frac{1}{2}\right)^2 - 1}$
$u = 3x - \frac{1}{2}$
$\dfrac{2u^2 - 3u + 1}{u^2 - 1}$
$\dfrac{(u-1)(2u-1)}{(u-1)(u+1)}$
$\dfrac{2u - 1}{u + 1}$
$\dfrac{2\left(3x - \frac{1}{2}\right) - 1}{3x - \frac{1}{2} + 1}$
$\dfrac{6x - 1 - 1}{3x + \frac{1}{2}}$
$\dfrac{6x - 2}{3x + \frac{1}{2}} \cdot \dfrac{2}{2}$
$\dfrac{12x - 4}{6x + 1}$

Exercise 5-1

1. $\dfrac{\sin\theta}{\tan\theta}$
$\dfrac{\sin\theta}{\dfrac{\sin\theta}{\cos\theta}}$
$\sin\theta \cdot \dfrac{\cos\theta}{\sin\theta}$
$\cos\theta$

5. $\cot^2\theta\sin^2\theta$
$\dfrac{\cos^2\theta}{\sin^2\theta} \cdot \sin^2\theta$
$\cos^2\theta$

9. $\dfrac{(\sec\theta - 1)(\sec\theta + 1)}{\sin^2\theta}$
$\dfrac{\sec^2\theta - 1}{\sin^2\theta}$
$\dfrac{\tan^2\theta}{\sin^2\theta}$ $\tan^2\theta + 1 = \sec^2\theta$, so $\tan^2\theta = \sec^2\theta - 1$
$\tan^2\theta\,\dfrac{1}{\sin^2\theta}$
$\dfrac{\sin^2\theta}{\cos^2\theta} \cdot \dfrac{1}{\sin^2\theta}$
$\dfrac{1}{\cos^2\theta}$
$\sec^2\theta$

13. $\cos\theta(\sec\theta - \cos\theta)$
$\cos\theta\sec\theta - \cos^2\theta$
$\cos\theta\,\dfrac{1}{\cos\theta} - \cos^2\theta$
$1 - \cos^2\theta$
$\sin^2\theta$

17. $\dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{1}{\sin^2\theta}$
$\dfrac{\cos^2\theta - 1}{\sin^2\theta}$
$\dfrac{-\sin^2\theta}{\sin^2\theta}$
-1

21. $\sec^2\theta(\csc^2\theta - 1)$
$\sec^2\theta[(\cot^2\theta + 1) - 1]$
$\sec^2\theta\cot^2\theta$
$\dfrac{1}{\cos^2\theta} \cdot \dfrac{\cos^2\theta}{\sin^2\theta}$
$\dfrac{1}{\sin^2\theta}$
$\csc^2\theta$

25. $\sin x + \cos x\cot x$
$\sin x + \cos x\,\dfrac{\cos x}{\sin x}$
$\dfrac{\sin^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x}$
$\dfrac{\sin^2 x + \cos^2 x}{\sin x}$
$\dfrac{1}{\sin x}$
$\csc x$

29. $\sec x - \tan x\sin x$
$\sec x - \dfrac{\sin x}{\cos x} \cdot \sin x$
$\dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x}$
$\dfrac{1 - \sin^2 x}{\cos x}$
$\sin^2\theta + \cos^2\theta = 1$, so $\cos^2\theta = 1 - \sin^2\theta$.
$\dfrac{\cos^2 x}{\cos x}$
$\cos x$

33. $\csc\theta + \cot\theta$
$\dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$
$\dfrac{1 + \cos\theta}{\sin\theta}$

37. $\dfrac{1 + \csc\theta}{1 + \sec\theta}$
$\dfrac{1 + \dfrac{1}{\sin\theta}}{1 + \dfrac{1}{\cos\theta}}$
$\dfrac{\dfrac{\sin\theta + 1}{\sin\theta}}{\dfrac{\cos\theta + 1}{\cos\theta}}$
$\dfrac{\sin\theta + 1}{\sin\theta} \cdot \dfrac{\cos\theta}{\cos\theta + 1}$
$\dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{\sin\theta + 1}{\cos\theta + 1}$
$\cot\theta\left(\dfrac{1 + \sin\theta}{1 + \cos\theta}\right)$

41. $\sec y - \cos y$
$\dfrac{1}{\cos y} - \cos y$
$\dfrac{1 - \cos^2 y}{\cos y}$
$\dfrac{\sin^2 y}{\cos y}$
$\dfrac{\sin y}{\cos y} \cdot \sin y$
$\tan y\sin y$

45. $\dfrac{1}{\sec\theta - \cos\theta}$
$\dfrac{1}{\dfrac{1}{\cos\theta} - \cos\theta}$
$\dfrac{1}{\dfrac{1 - \cos^2\theta}{\cos\theta}}$
$\dfrac{1}{\dfrac{\sin^2\theta}{\cos\theta}}$
$\dfrac{\cos\theta}{\sin^2\theta}$
$\dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{1}{\sin\theta}$
$\cot\theta\csc\theta$

49. $\dfrac{\tan^2\theta}{\sec\theta - 1}$
$\dfrac{\sec^2\theta - 1}{\sec\theta - 1}$
$\tan^2\theta + 1 = \sec^2\theta$, so $\tan^2\theta = \sec^2\theta - 1$.
$\dfrac{(\sec\theta - 1)(\sec\theta + 1)}{\sec\theta - 1}$
$\sec\theta + 1$

53. $\dfrac{\sin x}{1 + \cos x}$
$\dfrac{\sin x}{1 + \cos x} \cdot \dfrac{1 - \cos x}{1 - \cos x}$
$\dfrac{\sin x(1 - \cos x)}{1 - \cos^2 x}$
$\dfrac{\sin x(1 - \cos x)}{\sin^2 x}$
$\dfrac{1 - \cos x}{\sin x}$ Alternate solution in text

57. $\dfrac{1}{1 - \sin x} + \dfrac{1}{1 + \sin x}$
$\dfrac{(1 + \sin x) + (1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$
$\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$
$\dfrac{2}{1 - \sin^2 x} = \dfrac{2}{\cos^2 x} = 2\sec^2 x$

61. $\cot^2 x - \cos^2 x$

$\dfrac{\cos^2 x}{\sin^2 x} - \cos^2 x$

$\dfrac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x}$

$\dfrac{\cos^2 x(1 - \sin^2 x)}{\sin^2 x}$

$\dfrac{\cos^2 x}{\sin^2 x}(1 - \sin^2 x)$

$\cot^2 x - \cos^2 x$

65. Left side:

$\dfrac{1 - \sin x}{1 + \sin x}$

$\dfrac{1 - \sin x}{1 + \sin x} \cdot \dfrac{1 - \sin x}{1 - \sin x}$

$\dfrac{1 - 2\sin x + \sin^2 x}{1 - \sin^2 x}$

$\dfrac{1 - 2\sin x + \sin^2 x}{\cos^2 x}$

$\dfrac{1}{\cos^2 x} - \dfrac{2\sin x}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x}$

$\sec^2 x - 2\dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\cos x} + \tan^2 x$

$\sec^2 x - 2\tan x \sec x + \tan^2 x$

Right side:

$(\tan x - \sec x)^2$

$\tan^2 x - 2\tan x \sec x + \sec^2 x$

69. $2\cos^2 y - 1$

$2\cos^2 y - (\sin^2 y + \cos^2 y)$

$\cos^2 y - \sin^2 y$

In problems 73 and 77 let $\theta = \frac{\pi}{3}$; $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{\pi}{3} = \frac{1}{2}$; $\tan\frac{\pi}{3} = \sqrt{3}$.

<u>Different values are used in the text for θ.</u>

73. $\sec\theta = \dfrac{1}{\csc\theta}$

$2 = \dfrac{1}{\frac{2}{\sqrt{3}}}$

$2 \neq \dfrac{\sqrt{3}}{2}$

77. $\csc\theta + \sec\theta\cot\theta = 2$

$\dfrac{2}{\sqrt{3}} + 2 \cdot \dfrac{1}{\sqrt{3}} = 2$

$\dfrac{4}{\sqrt{3}} \neq 2$

81. (a) $(1 - \csc^2\theta)(1 - \sec^2\theta) = 1$

$\theta = \frac{\pi}{6}$: $(1 - \csc^2\frac{\pi}{6})(1 - \sec^2\frac{\pi}{6})$

$(1 - 2^2)(1 - (\frac{2}{\sqrt{3}})^2)$

$(-3)(1 - \frac{4}{3})$

$(-3)(-\frac{1}{3}) = 1$

(b) $\theta = \frac{\pi}{4}$: $(1 - \csc^2\frac{\pi}{4})(1 - \sec^2\frac{\pi}{4}) = 1$

$(1 - (\sqrt{2})^2)(1 - (\sqrt{2})^2)$

$(1 - 2)(1 - 2)$

1

(c) Yes: $(1 - \csc^2\theta)(1 - \sec^2\theta) = 1$

$(-\cot^2\theta)(-\tan^2\theta)$

$\dfrac{1}{\tan^2\theta}\tan^2\theta$

1

Exercise 5-2

1. $\sin 18° = \cos(90° - 18°) = \cos 72°$

5. $\sec\frac{\pi}{3} = \csc(\frac{\pi}{2} - \frac{\pi}{3}) = \csc\frac{\pi}{6}$

9. $\sec(-\frac{3\pi}{4}) = \csc\left(\frac{\pi}{2} - (-\frac{3\pi}{4})\right) = \csc\frac{5\pi}{4}$

13. $\cos 20° \csc 70° = \cos 20° \sec 20° = \cos 20° \cdot \dfrac{1}{\cos 20°} = 1$

17. $\tan^2 8° - \csc^2 82° = \tan^2 8° - \sec^2 8° = -1$

Since $\tan^2\theta + 1 = \sec^2\theta$, $\tan^2\theta - \sec^2\theta = -1$.

21. $\sec\frac{\pi}{6}\sin\frac{\pi}{3} = \csc\frac{\pi}{3}\sin\frac{\pi}{3} = 1$

25. $\sec^2\frac{\pi}{3} - \cot^2\frac{\pi}{6}$

$\sec^2\frac{\pi}{3} - \tan^2\frac{\pi}{3} = 1$

Since $\tan^2\theta + 1 = \sec^2\theta$, $\sec^2\theta - \tan^2\theta = 1$.

29. $\sin\frac{5\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$ $\qquad \sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

33. $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$ $\qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

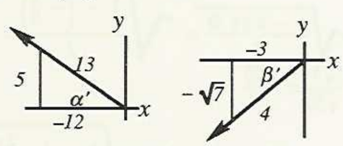
$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

37. $\tan 75° = \tan(45° + 30°) = \dfrac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$ $\qquad \tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \dfrac{1 + \frac{\sqrt{3}}{3}}{1 - 1\left(\frac{\sqrt{3}}{3}\right)} \cdot \dfrac{3}{3} = \dfrac{3 + \sqrt{3}}{3 - \sqrt{3}} \cdot \dfrac{3 + \sqrt{3}}{3 + \sqrt{3}} = \dfrac{12 + 6\sqrt{3}}{6} = 2 + \sqrt{3}$

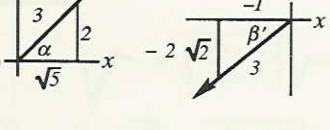
41. $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$= \dfrac{-\frac{5}{12} - \frac{\sqrt{7}}{3}}{1 + \left(-\frac{5}{12}\right)\left(\frac{\sqrt{7}}{3}\right)} = \dfrac{\frac{-15 - 12\sqrt{7}}{36}}{1 - \frac{5\sqrt{7}}{36}} \cdot \dfrac{36}{36} = \dfrac{-15 - 12\sqrt{7}}{36 - 5\sqrt{7}}$



45. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

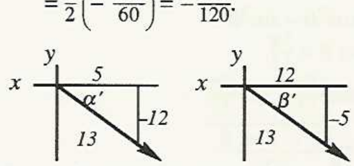
$= \frac{\sqrt{5}}{3}\left(-\frac{1}{3}\right) + \frac{2}{3}\left(-\frac{2\sqrt{2}}{3}\right) = \dfrac{-\sqrt{5} - 4\sqrt{2}}{9}$



49. $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

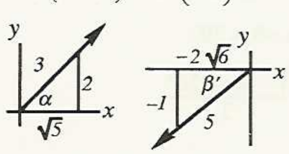
$= \dfrac{-\frac{12}{5} - \left(-\frac{5}{12}\right)}{1 + \left(-\frac{12}{5}\right)\left(-\frac{5}{12}\right)} = \dfrac{-\frac{12}{5} + \frac{5}{12}}{1 + 1} = \dfrac{\frac{-144 + 25}{60}}{2}$

$= \frac{1}{2}\left(-\frac{119}{60}\right) = -\frac{119}{120}.$



53. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{2}{3}\left(-\frac{2\sqrt{6}}{5}\right) - \frac{\sqrt{5}}{3}\left(-\frac{1}{5}\right) = \dfrac{-4\sqrt{6} + \sqrt{5}}{15}$



57. $\sin(\pi - \theta) = \sin\pi\cos\theta - \cos\pi\sin\theta$
$= 0\cos\theta - (-1)\sin\theta$
$= \sin\theta$

61. $\tan(\pi - \theta) = \dfrac{\tan\pi - \tan\theta}{1 + \tan\pi\tan\theta}$
$= \dfrac{0 - \tan\theta}{1 + 0\tan\theta}$
$= -\tan\theta$

65. $\tan(\theta + \pi) = \dfrac{\tan\theta + \tan\pi}{1 - \tan\theta\tan\pi}$
$= \dfrac{\tan\theta + 0}{1 - \tan\theta\,(0)}$
$= \tan\theta$

69. $\frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\frac{1}{2}[\cos\alpha\cos\beta + \sin\alpha\sin\beta - (\cos\alpha\cos\beta - \sin\alpha\sin\beta)]$
$\frac{1}{2}[2\sin\alpha\sin\beta] = \sin\alpha\sin\beta$

73. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ — This was shown true in the text.
$\cos(\alpha + (-\beta)) = \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta)$ — Replace β by $-\beta$. This is valid since the identity is true for all angles and α.
$\cos(\alpha - \beta) = \cos\alpha\cos\beta - \sin\alpha\,[-\sin\beta]$ — $\alpha + (-\beta) = \alpha - \beta$; $\cos(-\theta) = \cos\theta$; $\sin(-\theta) = -\sin\theta$.
$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. — This statement is true since the preceding statements are true.

77. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ — Identity [3].
$\sin(\alpha + (-\beta)) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta)$
$\sin(\alpha - \beta) = \sin\alpha\cos\beta + \cos\alpha\,[-\sin\beta]$ — Cosine is an even function, sine is odd.
$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$ — This is identity [4].

81. $\csc\left(\frac{\pi}{2} - \theta\right) = \dfrac{1}{\sin\left(\frac{\pi}{2} - \theta\right)} = \dfrac{1}{\cos\theta} = \sec\theta$

Exercise 5-3

1. $2\sin\frac{\pi}{4}\cos\frac{\pi}{4}$
$2\sin\alpha\cos\alpha = \sin 2\alpha$
Let $\alpha = \frac{\pi}{4}$, so $2\alpha = \frac{\pi}{2}$, and $\sin 2\alpha$ is $\sin\frac{\pi}{2}$.

5. $1 - 2\sin^2\frac{\pi}{10}$
$1 - 2\sin^2\alpha = \cos 2\alpha$
Let $\alpha = \frac{\pi}{10}$, so $2\alpha = \frac{\pi}{5}$, so $\cos 2\alpha = \cos\frac{\pi}{5}$.

9. $2\sin 6\theta\cos 6\theta$
$2\sin\alpha\cos\alpha = \sin 2\alpha$ — Let $\alpha = 6\theta$.
$\sin 2\alpha$ becomes $\sin 12\theta$.

13. $\dfrac{10\tan 3\theta}{1 - \tan^2 3\theta}$
$\dfrac{2\tan\alpha}{1 - \tan^2\alpha} = \tan 2\alpha$ — Identity.
$\dfrac{10\tan\alpha}{1 - \tan^2\alpha} = 5\tan 2\alpha$ — Multiply each member by 5.
Let $\alpha = 3\theta$, so $2\alpha = 6\theta$. Thus $5\tan 2\alpha$ is $5\tan 6\theta$.

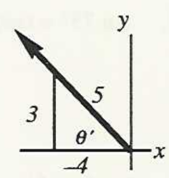
17. $3\cos^2 3\theta - 3\sin^2 3\theta$
$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$ — Identity.
$3\cos^2\alpha - 3\sin^2\alpha = 3\cos 2\alpha$
Multiply each member by 3. α represents 3θ, so 2α is 6θ.
Then $3\cos 2\alpha$ means $3\cos 6\theta$.

21. $\cos\frac{5\pi}{6} = \cos^2\theta - \sin^2\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$2\theta = \frac{5\pi}{6}$, so $\theta = \frac{5\pi}{12}$.
$\cos\frac{5\pi}{6} = \cos^2\frac{5\pi}{12} - \sin^2\frac{5\pi}{12}$.

25. $\sin 10° = \sqrt{\dfrac{1 - \cos\theta}{2}}$
$\sin\frac{\alpha}{2} = \sqrt{\dfrac{1 - \cos\alpha}{2}}$
$\frac{\alpha}{2} = 10°$, so $\alpha = \theta = 20°$.
$\sin 10° = \sqrt{\dfrac{1 - \cos 20°}{2}}$

29. $\tan\frac{2\pi}{5} = \sqrt{\dfrac{1 - \cos\theta}{1 + \cos\theta}}$
$\tan\frac{\alpha}{2} = \sqrt{\dfrac{1 - \cos\alpha}{1 + \cos\alpha}}$
$\frac{\alpha}{2} = \frac{2\pi}{5}$, so $\alpha = \theta = \frac{4\pi}{5}$, and
$\tan\frac{2\pi}{5} = \sqrt{\dfrac{1 - \cos\frac{4\pi}{5}}{1 + \cos\frac{4\pi}{5}}}$.

33. $\cos\theta = -\frac{4}{5}$, $\frac{\pi}{2} < \theta < \pi$
$\sin 2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{3}{5}\left(-\frac{4}{5}\right) = -\frac{24}{25}$
$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$
$\tan 2\theta = \dfrac{\sin 2\theta}{\cos 2\theta} = -\frac{24}{7}$



37. $\sec\theta = -\frac{5}{2}$, $\pi < \theta < \frac{3\pi}{2}$: $\cos\theta = -\frac{2}{5}$
$\frac{\pi}{2} \le \frac{\theta}{2} \le \frac{3\pi}{4}$, ($\frac{\theta}{2}$ in qII) so $\sin\frac{\theta}{2} > 0$, $\cos\frac{\theta}{2} < 0$, $\tan\frac{\theta}{2} < 0$.
$\sin\frac{\theta}{2} = \sqrt{\dfrac{1 - \cos\theta}{2}} = \sqrt{\dfrac{1 - \left(-\frac{2}{5}\right)}{2}} = \sqrt{\frac{1}{2} \cdot \frac{7}{5}}$
$= \frac{\sqrt{7}}{\sqrt{10}} = \frac{\sqrt{70}}{10}$
$\cos\frac{\theta}{2} = -\sqrt{\dfrac{1 + \cos\theta}{2}} = -\sqrt{\dfrac{1 + \left(-\frac{2}{5}\right)}{2}} = -\sqrt{\frac{1}{2} \cdot \frac{3}{5}}$
$= -\frac{\sqrt{3}}{\sqrt{10}} = -\frac{\sqrt{30}}{10}$
$\tan\frac{\theta}{2} = -\sqrt{\dfrac{1 - \cos\theta}{1 + \cos\theta}} = -\sqrt{\dfrac{1 - \left(-\frac{2}{5}\right)}{1 + \left(-\frac{2}{5}\right)}}$
$= -\sqrt{\frac{7}{10} \div \frac{3}{10}} = -\sqrt{\frac{7}{3}} = -\frac{\sqrt{7}}{\sqrt{3}} = -\frac{\sqrt{21}}{3}$

41. $15°$, or $\frac{\pi}{12}$

$$\sin 15° = \sqrt{\frac{1-\cos 30°}{2}} = \sqrt{\tfrac{1}{2}\left(1-\tfrac{\sqrt{3}}{2}\right)} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$\cos 15° = \sqrt{\frac{1+\cos 30°}{2}} = \sqrt{\tfrac{1}{2}\left(1+\tfrac{\sqrt{3}}{2}\right)} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\tan 15° = \frac{1-\cos 30°}{\sin 30°} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} \cdot \frac{2}{2} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$

45. $\cos 37.5° = \cos(15° + 22.5°) = \cos 15° \cos 22.5° - \sin 15° \sin 22.5°$

$$= \frac{\sqrt{2+\sqrt{3}}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} - \frac{\sqrt{2-\sqrt{3}}}{2} \cdot \frac{\sqrt{2-\sqrt{2}}}{2} = \frac{\sqrt{4+\sqrt{6}+2\sqrt{3}+2\sqrt{2}} - \sqrt{4+\sqrt{6}-2\sqrt{3}-2\sqrt{2}}}{4}$$

Note: A simpler solution is as follows: Use $\cos 75° = \cos(30° + 45°)$ and expand. This gives $\cos 75° = \frac{\sqrt{6}-\sqrt{2}}{4}$. Use this value in the identity $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$, with $\alpha = 75°$. This produces the solution $\frac{1}{4}\sqrt{2\sqrt{6}-2\sqrt{2}+8}$.

49.
$\sin 2\theta + 1$
$2 \sin\theta \cos\theta + 1$
$2 \sin\theta \cos\theta + \sin^2\theta + \cos^2\theta$
$\sin^2\theta + 2 \sin\theta \cos\theta + \cos^2\theta$
$(\sin\theta + \cos\theta)^2$
(Alternate solution in text)

53.
$\frac{1+\cos 2\theta}{1-\cos 2\theta}$
$\frac{1+(2\cos^2\theta - 1)}{1-(1-2\sin^2\theta)}$
$\frac{2\cos^2\theta}{2\sin^2\theta}$
$\frac{\cos^2\theta}{\sin^2\theta}$
$\cot^2\theta$

57.
$\sin 2\theta - 4\sin^3\theta \cos\theta$
$2\sin\theta\cos\theta - 4\sin^3\theta\cos\theta$
$2\sin\theta\cos\theta(1-2\sin^2\theta)$
$\sin 2\theta \cos 2\theta$
(Alternate solution in text)

61.
$\tan 2\theta = \frac{2(\tan\theta + \tan^3\theta)}{1-\tan^4\theta}$
$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\tan\theta(1+\tan^2\theta)}{(1-\tan^2\theta)(1+\tan^2\theta)}$
$= \frac{2\tan\theta}{1-\tan^2\theta}$

65.
$\frac{1-\tan^2\theta}{1+\tan^2\theta}$
$\frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$
$\frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$
$\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$
$\frac{\cos 2\theta}{1}$
$\cos 2\theta$

69.
$\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}$
$\left(\pm\sqrt{\frac{1+\cos\theta}{2}}\right)^2\left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right)^2$
$\frac{1+\cos\theta}{2} \cdot \frac{1-\cos\theta}{2}$
$\frac{1-\cos^2\theta}{4}$
$\frac{\sin^2\theta}{4}$

73.
$\sin^2\frac{\theta}{2} = \frac{\csc\theta - \cot\theta}{2\csc\theta}$
$\sin^2\frac{\theta}{2} = \frac{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}}{2\,\frac{1}{\sin\theta}} \cdot \frac{\sin\theta}{\sin\theta}$
$\left(\sqrt{\frac{1-\cos\theta}{2}}\right)^2 = \frac{1-\cos\theta}{2}$
$\frac{1-\cos\theta}{2}$

77.
$2\cos^2\frac{\theta}{2} - \cos\theta = 1$
$2\left(\sqrt{\frac{1+\cos\theta}{2}}\right)^2 - \cos\theta$
$2\,\frac{1+\cos\theta}{2} - \cos\theta$
$1 + \cos\theta - \cos\theta$
1

81. We know that $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\sin^3\theta\cos\theta$ and $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$ from problem 80.

a. $\sin 5\theta$
$= \sin(\theta + 4\theta)$
$= \sin\theta\cos 4\theta + \cos\theta\sin 4\theta$
$= \sin\theta(8\cos^4\theta - 8\cos^2\theta + 1) + \cos\theta(4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta)$
$= 4\sin\theta\cos^2\theta - 8\sin^3\theta\cos^2\theta + 8\sin\theta\cos^4\theta - 8\sin\theta\cos^2\theta + \sin\theta$

We know $\cos^2\theta = 1 - \sin^2\theta$, so that $\cos^4\theta = (1-\sin^2\theta)^2 = 1 - 2\sin^2\theta + \sin^4\theta$.
Replace $\cos^2\theta$ and $\cos^4\theta$ in the equation above:
$= 4\sin\theta(1-\sin^2\theta) - 8\sin^3\theta(1-\sin^2\theta) + 8\sin\theta(1 - 2\sin^2\theta + \sin^4\theta)$
$\quad - 8\sin\theta(1-\sin^2\theta) + \sin\theta$
$= 4\sin\theta - 4\sin^3\theta - 8\sin^3\theta + 8\sin^5\theta + 8\sin\theta - 16\sin^3\theta + 8\sin^5\theta$
$\quad - 8\sin\theta + 8\sin^3\theta + \sin\theta$
$= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$

b. $\cos 5\theta$
$= \cos(\theta + 4\theta)$
$= \cos\theta\cos 4\theta - \sin\theta\sin 4\theta$
$= \cos\theta(8\cos^4\theta - 8\cos^2\theta + 1) - \sin\theta(4\cos^3\theta\sin\theta - 4\sin^3\theta\cos\theta)$
$= 8\cos^5\theta - 8\cos^3\theta + \cos\theta - 4\cos^3\theta\sin^2\theta + 4\sin^4\theta\cos\theta$
$= 8\cos^5\theta - 8\cos^3\theta + \cos\theta - 4\cos^3\theta(1-\cos^2\theta) + 4(1-\cos^2\theta)^2\cos\theta$
$= 8\cos^5\theta - 8\cos^3\theta + \cos\theta - 4\cos^3\theta + 4\cos^5\theta + 4\cos\theta - 8\cos^3\theta + 4\cos^5\theta$
$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

85. (a) Problem 41 shows that $\sin 15° = \frac{\sqrt{2-\sqrt{3}}}{2}$.

(b) $\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

(c) $\frac{\sqrt{6}-\sqrt{2}}{4} \stackrel{?}{=} \frac{\sqrt{2-\sqrt{3}}}{2}$

$\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2 \quad \left(\frac{\sqrt{2-\sqrt{3}}}{2}\right)^2$ Square both values.

$\left(\frac{(\sqrt{6}-\sqrt{2})^2}{16}\right) \quad \frac{2-\sqrt{3}}{4}$

$\frac{6-2\sqrt{12}+2}{16}$

$\frac{8-4\sqrt{3}}{16}$

$\frac{4(2-\sqrt{3})}{16}$

$\frac{2-\sqrt{3}}{4}$

89. $\cos \theta_2 = \cos\frac{\theta}{2} = \frac{8}{9}$; $\cos \theta = \frac{8}{x}$

$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$ The angles are acute, so we need the positive value.

$\frac{8}{9} = \sqrt{\frac{1+\frac{8}{x}}{2}}$ Substitute values.

$\frac{64}{81} = \frac{1+\frac{8}{x}}{2}$ Square both members.

$128 = 81\left(1+\frac{8}{x}\right)$ If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

$128 = 81 + \frac{648}{x}$

$47 = \frac{648}{x}$

$x = \frac{648}{47} = 13\frac{37}{47}$

93.

$\cos 2\alpha + \cos 2\beta$	=	$2\cos(\alpha+\beta) \cdot \cos(\alpha-\beta)$
$(2\cos^2\alpha - 1) + (2\cos^2\beta - 1)$		$2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$
$2\cos^2\alpha + 2\cos^2\beta - 2$		$2(\cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta)$
		$2[\cos^2\alpha\cos^2\beta - (1-\cos^2\alpha)(1-\cos^2\beta)]$
		$2[\cos^2\alpha\cos^2\beta - (1 - \cos^2\beta - \cos^2\alpha + \cos^2\alpha\cos^2\beta)]$
		$2[-1 + \cos^2\beta + \cos^2\alpha]$
		$-2 + 2\cos^2\beta + 2\cos^2\alpha$

Exercise 5-4

1. $\tan\theta + 1 = 0$

$\tan\theta = -1$

$\theta' = \tan^{-1} 1 = \frac{\pi}{4}\,(45°)$

tangent is negative in qII and qIV, so $\theta = \pi - \theta' = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $(180° - 45° = 135°)$

or $2\pi - \theta' = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ $(360° - 45° = 315°)$.

5. $\sqrt{3}\tan\theta - 1 = 0$

$\tan\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$;

$\theta' = \tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{6}\,(30°)$.

$\tan\theta > 0$ in qI and qIII, so $\theta = \theta' = \frac{\pi}{6}\,(30°)$ and

$\pi + \theta' = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ $(180° + 30° = 210°)$.

9. $3\sin^2\theta - 3 = 0$

$\sin^2\theta - 1 = 0$ Divide each member by 3.

$\sin^2\theta = 1$

$\sin\theta = \pm 1$

$\sin\theta = 1$ at $\frac{\pi}{2}\,(90°)$ and $\sin\theta = -1$ at $\frac{3\pi}{2}\,(270°)$.

13. $(\cos\theta - 1)(\sin\theta + 1) = 0$

$\cos\theta - 1 = 0$ or $\sin\theta + 1 = 0$ Zero product property.

$\cos\theta = 1$ or $\sin\theta = -1$

$0\,(0°)$ $\quad \frac{3\pi}{2}\,(270°)$

17. $\sin^2\theta - \sin\theta = 0$

$\sin\theta(\sin\theta - 1) = 0$ $\quad a^2 - a = a(a-1)$

$\sin\theta = 0$ or $\sin\theta - 1 = 0$

$0\,(0°)$ and $\pi\,(180°)$ $\quad \sin\theta = 1$

$\frac{\pi}{2}\,(90°)$

21. $2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$ Factor like $2u^2 + u - 1 = 0$.

$2\sin\theta - 1 = 0$ or $\sin\theta + 1 = 0$

$2\sin\theta = 1$ or $\sin\theta = -1$

$\sin\theta = \frac{1}{2}$ $\quad \frac{3\pi}{2}\,(270°)$

$\theta' = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$; $\sin\theta > 0$ in qI and qII.

$\theta = \theta' = \frac{\pi}{6}\,(30°)$ and $\pi - \frac{\pi}{6} = \frac{5\pi}{6}\,(150°)$

25. $2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ or $2\cos\theta - 1 = 0$

$0\,(0°), \pi\,(180°)$ $\quad \cos\theta = \frac{1}{2}$

$\frac{\pi}{3}\,(60°)$ and $\frac{5\pi}{3}\,(300°)$

(See problem 3.)

29. $\tan x \cot x = 0$

$\tan x = 0$ or $\cot x = 0$

$\frac{\sin x}{\cos x} = 0$ or $\frac{\cos x}{\sin x} = 0$

$\sin x = 0$ or $\cos x = 0$

$0\,(0°), \pi\,(180°)$ $\quad \frac{\pi}{2}\,(90°), \frac{3\pi}{2}\,(270°)$

33. $\tan x + \cot x = -2$

$\tan x + \frac{1}{\tan x} = -2$

$\tan^2 x + 1 = -2\tan x$ Multiply each member by $\tan x$; assume $\tan x \neq 0$.

$\tan^2 x + 2\tan x + 1 = 0$

$(\tan x + 1)^2 = 0$

$\tan x + 1 = 0$

$\tan x = -1$

$\frac{3\pi}{4}\,(135°)$ and $\frac{7\pi}{4}\,(315°)$ (Problem 1.)

37. $4\tan^2 x = 3\sec^2 x$

$4\tan^2 x = 3(\tan^2 x + 1)$

$4\tan^2 x = 3\tan^2 x + 3$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\theta' = \tan^{-1}\sqrt{3} = \frac{\pi}{3}\,(60°)$

Since $\tan\theta$ is both positive and negative there are solutions in each quadrant.

qI: $\theta = \theta' = \frac{\pi}{3}$ (60°)

qII: $\theta = \pi - \theta' = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ (180° − 60° = 120°)

qIII: $\theta = \pi + \theta' = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ (180° + 60° = 240°)

qIV: $\theta = 2\pi - \theta' = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ (360° − 60° = 300°)

41. $2\tan^2 x \sin x = \tan^2 x$

$2\tan^2 x \sin x - \tan^2 x = 0$

$\tan^2 x\,(2\sin x - 1) = 0$

$\tan^2 x = 0$	or	$2\sin x - 1 = 0$
$\tan x = 0$	or	$\sin x = \frac{1}{2}$
0 (0°), π (180°) (Prob. 19)		$\frac{\pi}{6}$ (30°) and $\frac{5\pi}{6}$ (150°) (Prob. 21.)

45. $\cot^2 x - 3\cot x - 2 = 0$

$a = 1$, $b = -3$, $c = -2$:

$\cot x = \frac{3 \pm \sqrt{9 - 4(-2)}}{2} = \frac{3 \pm \sqrt{17}}{2}$

$\cot x = \frac{3 + \sqrt{17}}{2}$	$\cot x = \frac{3 - \sqrt{17}}{2}$
$\tan x = \frac{2}{3 + \sqrt{17}}$	$\tan x = \frac{2}{3 - \sqrt{17}}$
$x' = \tan^{-1} \frac{2}{3 + \sqrt{17}}$	$x' = \tan^{-1}\left\lvert \frac{2}{3 + \sqrt{17}} \right\rvert$
$x' \approx 0.274$ (15.68°)	$x' = 1.059$ (60.68°)
$\tan x > 0$ in qI and qIII.	$\tan x < 0$ in qII and qIV.
$x = x' \approx 0.27$	$x = \pi - x' \approx \pi - 1.059 \approx 2.08$
$= \pi + x' \approx \pi + 0.274 \approx 3.42$	$= 2\pi - x' \approx 2\pi - 1.059 \approx 5.22$
$x° = x' \approx 15.7°$	$x° = 180° - x' \approx 180° - 60.7° \approx 119.3°$
$= 180° + x' \approx 180° + 15.7° = 195.7°$	$= 360° - x' \approx 360° - 60.7° \approx 299.3°$

49. $\tan x + 2\sec x = 3$

$\tan x = 3 - 2\sec x$

$(\tan x)^2 = (3 - 2\sec x)^2$ Since we are squaring both sides we must check all answers.

$\tan^2 x = 9 - 12\sec x + 4\sec^2 x$

$\sec^2 x - 1 = 9 - 12\sec x + 4\sec^2 x$

$3\sec^2 x - 12\sec x + 10 = 0$

$a = 3$, $b = -12$, $c = 10$: $\sec x = \frac{12 \pm \sqrt{144 - 120}}{6} = \frac{12 \pm 2\sqrt{6}}{6} = \frac{6 \pm \sqrt{6}}{3}$

$\sec x = \frac{6 + \sqrt{6}}{3}$	$\sec x = \frac{6 - \sqrt{6}}{3}$
$\cos x = \frac{3}{6 + \sqrt{6}} \approx 0.3551$	$\cos x = \frac{3}{6 - \sqrt{6}} \approx 0.8449$
$x = \cos^{-1} \frac{3}{6 + \sqrt{6}} \approx 1.208$ (69.2°)	$x = \cos^{-1} \frac{3}{6 - \sqrt{6}} \approx 0.564$ (32.3°)
$\cos x > 0$ in qI and qIV.	$\cos x > 0$ in qI and qIV.
$x = x' \approx 1.21$	$x = x' \approx 0.56$
$= 2\pi - x' \approx 2\pi - 1.208 \approx 5.08$	$= 2\pi - x' \approx 2\pi - 0.56 \approx 5.72$
$x° = x' \approx 69.2°$	$x° = x' \approx 32.3°$
$= 360° - x' \approx 360° - 69.2° \approx 290.8°$	$= 360° - x' \approx 360° - 32.3° \approx 327.7°$

As stated above we must check the solutions. 1.21 and 5.72 do not check:

$\tan(1.21) + 2\sec(1.21) \approx 8.3$, not 3, and

$\tan(5.72) + 2\sec(5.72) \approx 1.7$, not 3.

Thus the solutions are 5.08 (290.8°) and 0.56 (32.3°).

53. $\cot x = -\sqrt{3}$; $\tan x = -\frac{\sqrt{3}}{3}$

$x' = \tan^{-1} \frac{\sqrt{3}}{3} = \frac{\pi}{6}$ (30°).

Primary solutions are in qII and qIV: $\frac{5\pi}{6}$ (150°) and $\frac{11\pi}{6}$ (330°).

For all solutions we add $k\pi$ to these. Since the primary solutions differ by $k\pi$ ($k = 1$), we only need mention one primary solution to describe all solutions.

All solutions: $\frac{5\pi}{6} + k\pi$ (150° + k•180°).

57. $\tan x = 1$

$x = \tan^{-1} 1 = \frac{\pi}{4}$ (45°)

Primary solutions are in qI and qIII: $\frac{\pi}{4}$ (45°) and $\frac{5\pi}{4}$ (225°).

These differ by π (180°), so we can write all solutions with one of them: $\frac{\pi}{4} + k\pi$ (45° + k•180°).

61. $\sin \frac{x}{2} = \frac{\sqrt{3}}{2}$

$\left(\frac{x}{2}\right)' = \sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$ (60°)

Primary solutions to $\frac{x}{2}$ are in qI and qII. $\frac{\pi}{3}$ (60°) and $\frac{2\pi}{3}$ (120°).

All solutions:

$\frac{x}{2} = \frac{\pi}{3} + 2k\pi$ (60° + k•360°) and $\frac{2\pi}{3} + 2k\pi$ (120° + k•360°).

$x = \frac{2\pi}{3} + 4k\pi$ (120° + k•720°) and $\frac{4\pi}{3} + 4k\pi$ (240° + k•720°).

65. $3\cot 2x = \sqrt{3}$

$\cot 2x = \frac{\sqrt{3}}{3}$

$\tan 2x = \sqrt{3}$

$(2x)' = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$ (60°)

Primary solutions: $2x = \frac{\pi}{3}$ (60°) and $\frac{4\pi}{3}$ (240°)

All solutions:

$2x = \frac{\pi}{3} + k\pi$ (60° + k•180°) and $\frac{4\pi}{3} + k\pi$ (240° + k•180°)

$2x = \frac{\pi}{3} + k\pi$ (60° + k•180°) The second expression is redundant.

$x = \frac{\pi}{6} + k\frac{\pi}{2}$ (30° + k•90°)

69. $2\cos 2x + 1 = 0$

$\cos 2x = -\frac{1}{2}$

$(2x)' = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$ (60°)

Primary solutions: $2x = \frac{2\pi}{3}$ (120°) and $\frac{4\pi}{3}$ (240°)

All solutions:

$2x = \frac{2\pi}{3} + 2k\pi$ (120° + k•360°) and $\frac{4\pi}{3} + 2k\pi$ (240° + k•360)

$x = \frac{\pi}{3} + k\pi$ (60° + k•180°) and $\frac{2\pi}{3} + k\pi$ (120° + k•180)

73. $2\sin 2\theta = 1$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$ (30°)

Primary solutions:

$2\theta = \frac{\pi}{6} + 2k\pi$ (30° + k•360°) and $\frac{5\pi}{6} + 2k\pi$ (150° + k•360°)

All solutions:

$\theta = \frac{\pi}{12} + k\pi$ (15° + k•180°) and $\frac{5\pi}{12} + k\pi$ (75° + k•180°)

77. $\sqrt{3}\tan \frac{\theta}{4} = 1$

$\tan \frac{\theta}{4} = \frac{\sqrt{3}}{3}$

$\left(\frac{\theta}{4}\right)' = \tan^{-1} \frac{\sqrt{3}}{3} = \frac{\pi}{6}$ (30°)

Primary solutions: $\frac{\theta}{4} = \frac{\pi}{6}$ (30°) and $\frac{7\pi}{6}$ (210°)

All solutions:

$\frac{\theta}{4} = \frac{\pi}{6} + k\pi$ (30° + k•180°) and $\frac{7\pi}{6} + k\pi$ (210° + k•180°)

$\frac{\theta}{4} = \frac{\pi}{6} + k\pi$ (30° + k•180°)   The second expression is redundant.

$\theta = \frac{2\pi}{3} + 4k\pi$ (120° + k•720°)

81. $\sin 2\theta + \sin\theta = 0$
$2\sin\theta\cos\theta + \sin\theta = 0$
$\sin\theta(2\cos\theta + 1) = 0$

$\sin\theta = 0$	$2\cos\theta + 1 = 0$
0 (0°), π (180°)	$\cos\theta = -\frac{1}{2}$
	$\frac{2\pi}{3}$ (120°), $\frac{4\pi}{3}$ (240°)

85. $\sin\frac{\theta}{2} = \tan\frac{\theta}{2}$

$\sin\frac{\theta}{2} = \dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sin\frac{\theta}{2}$

$\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \sin\frac{\theta}{2} = 0$

$\sin\frac{\theta}{2}(\cos\frac{\theta}{2} - 1) = 0$

$\sin\frac{\theta}{2} = 0$	$\cos\frac{\theta}{2} = 1$
$\frac{\theta}{2} = 0$ (0°), π (180°)	$\frac{\theta}{2} = 0$ (0°)
$\theta = 0$ (0°), 2π (360°)	$\theta = 0$ (0°)
Primary solutions: 0 (0°)	

89. $\tan\frac{\theta}{2} = \cos\theta - 1$

$\pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \cos\theta - 1$

$\left(\pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}\right)^2 = (\cos\theta - 1)^2$ All solutions must be checked, since we square both members.

$\dfrac{1-\cos\theta}{1+\cos\theta} = \cos^2\theta - 2\cos\theta + 1$

$1 - \cos\theta = (\cos\theta + 1)(\cos^2\theta - 2\cos\theta + 1)$

$1 - \cos\theta = \cos^3\theta - \cos^2\theta - \cos\theta + 1$

$\cos^3\theta - \cos^2\theta = 0$

$\cos^2\theta(\cos\theta - 1) = 0$

$\cos^2\theta = 0$	$\cos\theta - 1 = 0$
$\cos\theta = 0$	$\cos\theta = 1$
$\frac{\pi}{2}$ (90°), $\frac{3\pi}{2}$ (270°)	0 (0°)

These results must be checked because we squared both members in the first step.

$\frac{\pi}{2}$ does not check: $\tan\dfrac{\frac{\pi}{2}}{2} = \cos\frac{\pi}{2} - 1$

$\tan\frac{\pi}{4} = \cos\frac{\pi}{2} - 1$

$1 = 0 - 1$

$1 \neq -1$

The rest of the solutions check.

Thus the solutions are 0 (0°) and $\frac{3\pi}{2}$ (270°).

93. If $B = 0.7$, $x = 2$, $y = -8$, find A to the nearest 0.01.
$-8 = 2\cos A\cos 0.7 - 4\cos A\sin 0.7 - 8\sin A$
$-8 = (2\cos 0.7)\cos A - (4\sin 0.7)\cos A - 8\sin A$
$-8 = 1.5297\cos A - 2.5769\cos A - 8\sin A$
$-8 = -1.0472\cos A - 8\sin A$
$8\sin A - 8 = -1.0472\cos A$
$\sin A - 1 = -0.1309\cos A$ Divide each member by 8.
$\sin A = 1 - 0.1309\cos A$
$(\sin A)^2 = (1 - 0.1309\cos A)^2$
$\sin^2 A = 1 - 0.2618\cos A + 0.017134\cos^2 A$
$1 - \cos^2 A = 1 - 0.2618\cos A + 0.017134\cos^2 A$
$0 = 1.0171\cos^2 A - 0.2618\cos A$
$0 = \cos A(1.0171\cos A - 0.2618)$
$\cos A = 0$ or $1.0171\cos A - 0.2618 = 0$
$A = \cos^{-1} 0$ or $1.0171\cos A = 0.2618$
$A = \frac{\pi}{2}$ or $\cos A = 0.25739$
$A \approx 1.310476103$

Thus A is $\frac{\pi}{2}$ or 1.31

Chapter 5 Review

1. $\dfrac{\cot\theta}{\cos\theta}$

$\dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{1}{\cos\theta}$

$\dfrac{1}{\sin\theta}$

$\csc\theta$

3. Left side:

$\dfrac{\tan^2\theta}{\sec^2\theta - 1}$

$\dfrac{\tan^2\theta}{\tan^2\theta}$

1

Right side:

$\sin^4\theta\csc^4\theta$

$\sin^4\theta\dfrac{1}{\sin^4\theta}$

1

5. $\dfrac{\csc\theta\tan\theta}{\sin\theta}$

$\csc\theta\tan\theta\dfrac{1}{\sin\theta}$

$\dfrac{1}{\sin\theta}\cdot\dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\sin\theta}$

$\dfrac{1}{\cos\theta}\cdot\dfrac{1}{\sin\theta}$

$\csc\theta\sec\theta$

7. $\csc\theta - \sec\theta$

$\dfrac{1}{\sin\theta} - \dfrac{1}{\cos\theta}$

$\dfrac{\cos\theta - \sin\theta}{\sin\theta\cos\theta}$

9. $\dfrac{\sec\theta}{\tan\theta - \cot\theta}$

$\dfrac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}}$

$\dfrac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta - \cos^2\theta}{\cos\theta\sin\theta}}$

$\dfrac{1}{\cos\theta}\cdot\dfrac{\cos\theta\sin\theta}{\sin^2\theta - \cos^2\theta}$

$\dfrac{\sin\theta}{\sin^2\theta - \cos^2\theta}$

11. $\dfrac{\sec^2\theta - 1}{\sec^2\theta}$

$\dfrac{\sec^2\theta}{\sec^2\theta} - \dfrac{1}{\sec^2\theta}$

$1 - \cos^2\theta$

$\sin^2\theta$

13. $\csc x - \tan x\cot x$

$\dfrac{1}{\sin x} - \tan x\dfrac{1}{\tan x}$

$\csc x - 1$

15. $\csc^2 x\sec^2 x(\cos^2 x - \sin^2 x)$

$\csc^2 x\sec^2 x\cos^2 x - \csc^2 x\sec^2 x\sin^2 x$

$\csc^2 x\dfrac{1}{\cos^2 x}\cos^2 x - \dfrac{1}{\sin^2 x}\sec^2 x\sin^2 x$

$\csc^2 x - \sec^2 x$

17. $\sec x(\sin x - \cos x)$

$\sec x\sin x - \sec x\cos x$

$\dfrac{1}{\cos x}\sin x - \dfrac{1}{\cos x}\cos x$

$\tan x - 1$

19. $\dfrac{1}{1+\csc x} + \dfrac{1}{1-\csc x}$

$\dfrac{(1-\csc x) + (1+\csc x)}{(1+\csc x)(1-\csc x)}$

$\dfrac{2}{1-\csc^2 x}$

$\dfrac{2}{-(\csc^2 x - 1)}$

$\dfrac{2}{-\cot^2 x}$

$-2\tan^2 x$

21. $\tan^4 x + \tan^2 x$

$\tan^2 x(\tan^2 x + 1)$

$\tan^2 x\cdot\sec^2 x$

$\frac{1}{\cot^2 x} \cdot \sec^2 x$

$\frac{\sec^2 x}{\cot^2 x}$ Alternate solution in text

23. $\sin\theta + \cos\theta = 1$

Let $\theta = \frac{\pi}{4}$: $\sin\frac{\pi}{4} + \cos\frac{\pi}{4} =$

$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$; $\sqrt{2} \neq 1$

25. $\frac{1}{\cot\theta - \csc\theta} = \sec\theta$

Let $\theta = \frac{\pi}{4}$: $\frac{1}{1 - \sqrt{2}} \neq \sqrt{2}$

27. $\tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$

$= -\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= -\frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}}$

$= -\frac{1 - \frac{\sqrt{3}}{3}}{1 + 1\left(\frac{\sqrt{3}}{3}\right)} \cdot \frac{3}{3}$

$= -\frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$

$= -\frac{12 - 6\sqrt{3}}{6} = -2 + \sqrt{3}$

29. $\tan(-15°) = -\tan 15° = -\tan(45° - 30°)$

$= -\frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$= -\frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= -\frac{\sqrt{3} - 1}{\sqrt{3} + 1} \cdot \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$

$= -\frac{3 - 2\sqrt{3} + 1}{3 - 1} =$

$-\frac{4 - 2\sqrt{3}}{2} = -2 + \sqrt{3}$

31. a. $\cos(\alpha + \beta) =$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta =$

$-\frac{\sqrt{3}}{2}\left(-\frac{\sqrt{15}}{4}\right) - \frac{1}{2}\left(-\frac{1}{4}\right) =$

$\frac{3\sqrt{5}}{8} + \frac{1}{8} = \frac{3\sqrt{5} + 1}{8}$

b. $\tan(\alpha - \beta) =$

$\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} =$

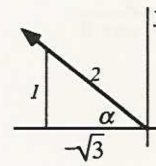
$\frac{-\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{15}}}{1 + \left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{15}}\right)} \cdot \frac{\sqrt{45}}{\sqrt{45}} =$

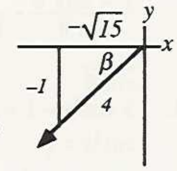
$\frac{-\sqrt{15} - \sqrt{3}}{\sqrt{45} - 1} = \frac{-\sqrt{15} - \sqrt{3}}{3\sqrt{5} - 1} \cdot \frac{3\sqrt{5} + 1}{3\sqrt{5} + 1} =$

$\frac{-3\sqrt{75} - \sqrt{15} - 3\sqrt{15} - \sqrt{3}}{45 - 1} =$

$\frac{-15\sqrt{3} - 4\sqrt{15} - \sqrt{3}}{44} = \frac{-16\sqrt{3} - 4\sqrt{15}}{44}$

$\frac{-4\sqrt{3} - \sqrt{15}}{11}$





33. $\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta} =$

$\frac{\frac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} + \frac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} + \frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}} =$

$\frac{\cot\beta + \cot\alpha}{\cot\alpha\cot\beta + 1}$

35. $\sin\left(\frac{\pi}{4} - \theta\right) = \sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\sin\theta$

$= \frac{\sqrt{2}}{2}\cos\theta - \frac{\sqrt{2}}{2}\sin\theta = \frac{\sqrt{2}}{2}(\cos\theta - \sin\theta)$

37. $\tan(\theta + \pi) = \frac{\tan\theta + \tan\pi}{1 - \tan\theta\tan\pi} =$

$\frac{\tan\theta + 0}{1 - 0 \cdot \tan\theta} = \tan\theta$

$\cot(\theta + \pi) = \frac{1}{\tan(\theta + \pi)} = \frac{1}{\tan\theta} = \cot\theta$

39. $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\sin\theta = 2\sin 5\pi\cos 5\pi$; $\alpha = 5\pi$ so $2\alpha = \theta = 10\pi$

41. $\cos 2\beta = 1 - 2\sin^2\beta$

$\cos 24° = 1 - 2\sin^2\theta$; $2\beta = 24°$ so $\beta = \theta = 12°$

43. $\tan 2a = \frac{2\tan a}{1 - \tan^2 a}$

$2\tan 2a = \frac{4\tan a}{1 - \tan^2 a}$; $a = 4\theta$ so $2a = 8\theta$;

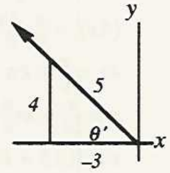
$2\tan 8\theta = \frac{4\tan 4\theta}{1 - \tan^2 4\theta}$

The result is $2\tan 8\theta$.

45. (a) $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(-\frac{3}{5}\right)^2 - 1 = \frac{18}{25} - 1 = -\frac{7}{25}$

(b) $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2\left(-\frac{4}{3}\right)}{1 - \left(-\frac{4}{3}\right)^2}$

$= \frac{-\frac{8}{3}}{1 - \frac{16}{9}} \cdot \frac{9}{9} = \frac{-24}{9 - 16} = \frac{24}{7}.$



47. $\sin 2x - \cos x = \cos x(2\sin x - 1)$

$2\sin x\cos x - \cos x$

$\cos x(2\sin x - 1)$

49. $\frac{\cos 2x}{2 - 4\sin^2 x}$

$\frac{\cos 2x}{2(1 - 2\sin^2 x)}$

$\frac{\cos 2x}{2(\cos 2x)} = \frac{1}{2}$

51. $\sin 2x - \cos 2x$

$2\sin x\cos x - (2\cos^2 x - 1)$

$2\sin x\cos x - 2\cos^2 x + 1$

$2\cos x(\sin x - \cos x) + 1$

53. $\cos(-15°) = \cos 15°$ (cosine is an even function)

$= \cos\frac{30°}{2} = \sqrt{\frac{1 + \cos 30°}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} =$

$\sqrt{\frac{2 + \sqrt{3}}{4}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$

55. $\cos\frac{3\pi}{8} = \cos\frac{\frac{3\pi}{4}}{2} = \sqrt{\frac{1 + \cos\frac{3\pi}{4}}{2}} = \sqrt{\frac{1 + \left(-\frac{\sqrt{2}}{2}\right)}{2}}$

$= \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

57. Given $\sin x = -\frac{2}{3}$, $\pi < x < \frac{3\pi}{2}$. Find (a) $\cos\frac{x}{2}$; (b) $\tan\frac{x}{2}$.

$\pi < x < \frac{3\pi}{2}$, so x terminates in quadrant III; also, $\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$, so $\frac{x}{2}$ terminates in quadrant II, where cosine is negative and tangent is negative.

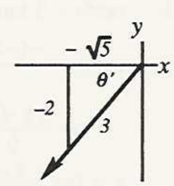
$\cos\frac{x}{2} = -\sqrt{\frac{1 + \cos x}{2}} = -\sqrt{\frac{1 + \left(-\frac{\sqrt{5}}{3}\right)}{2}} = -\sqrt{\frac{3 - \sqrt{5}}{6}}$

$\sec\frac{x}{2} = -\sqrt{\frac{6}{3 - \sqrt{5}}}$

$\tan\frac{x}{2} = \frac{1 - \cos\theta}{\sin\theta} = \frac{1 - \left(\frac{-\sqrt{5}}{3}\right)}{-\frac{2}{3}} \cdot \frac{3}{3}$

$= \frac{3 + \sqrt{5}}{-2} = -\frac{3 + \sqrt{5}}{2}$

$\cot\frac{x}{2} = -\frac{2}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}} = \frac{-3 + \sqrt{5}}{2}$



59. $\cot\frac{\theta}{2} = \dfrac{1}{\tan\frac{\theta}{2}} = \dfrac{1}{\dfrac{\sin\theta}{1+\cos\theta}} = \dfrac{1+\cos\theta}{\sin\theta}$

61. $\tan\frac{\theta}{2}\csc^2\frac{\theta}{2}$

$\dfrac{1-\cos\theta}{\sin\theta}\bullet\dfrac{1}{\sin^2\frac{\theta}{2}}$

$\dfrac{1-\cos\theta}{\sin\theta}\bullet\dfrac{1}{\dfrac{1-\cos\theta}{2}}$

$\dfrac{1-\cos\theta}{\sin\theta}\bullet\dfrac{2}{1-\cos\theta}$

$\dfrac{2}{\sin\theta}$

63. $3\cot^2 x - 1 = 0$

$\cot^2 x = \frac{1}{3}$

$\cot x = \pm\frac{1}{\sqrt{3}}$

$\tan x = \pm\sqrt{3}$

$x' = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$x = \frac{\pi}{3}$

65. $(4\sin^2 x - 1)(\sec x - 2) = 0$

$4\sin^2 x - 1 = 0$

$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$\frac{\pi}{6}$

$\sec x - 2 = 0$

$\sec x = 2$

$\cos x = \frac{1}{2}$

$\frac{\pi}{3}$

67. $\sec^2\theta - 4 = 0$

$\sec^2\theta = 4$

$\sec\theta = \pm 2$

$\cos\theta = \pm\frac{1}{2}$

60°, 120°, 240°, 300°

69. $2\sin\theta - \csc\theta + 1 = 0$

$2\sin\theta - \dfrac{1}{\sin\theta} + 1 = 0$

$2\sin^2\theta - 1 + \sin\theta = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$

30°, 150°

$\sin\theta = -1$

270°

71. $2\sin^2\theta - 3\cos\theta = 3$

$2(1-\cos^2\theta) - 3\cos\theta = 3$

$2 - 2\cos^2\theta - 3\cos\theta = 3$

$2\cos^2\theta + 3\ co\ s\theta + 1 = 0$

$(2\cos\theta + 1)(\cos\theta + 1) = 0$

$2\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{2}$

120°, 240°

$\cos\theta + 1 = 0$

$\cos\theta = -1$

180°

$\boxed{\text{If } ax^2+bx+c=0 \text{ then } x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}.}$

73. $\tan^2 x - 3\tan x - 3 = 0$; $a = 1, b = -3, c = -3$

$\tan x = \dfrac{-(-3)\pm\sqrt{(-3)^2-4(1)(-3)}}{2(1)} = \dfrac{3\pm\sqrt{9-(-12)}}{2}$

$= \dfrac{3\pm\sqrt{21}}{2}$

$x = \tan^{-1}\frac{3-\sqrt{21}}{2} \approx -0.669$

$x = \tan^{-1}\frac{3+\sqrt{21}}{2} \approx 1.31$

$x \approx -0.67 + k\pi$ or $1.31 + k\pi$

75. $\sec^2 x - \sec x = 2$

$\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x - 2 = 0$

$\sec x = 2$

$\cos x = \frac{1}{2}$

$x' = \frac{\pi}{3}, \frac{5\pi}{3}$

$\sec x + 1 = 0$

$\sec x = -1$

$\cos x = -1$

$x' = \pi$

$x = \frac{\pi}{3} + 2k\pi,\ \frac{5\pi}{3} + 2k\pi,\ \pi + 2k\pi$

77. $2\cos\frac{x}{2} - \sqrt{3} = 0$

$\cos\frac{x}{2} = \frac{\sqrt{3}}{2}$

$\frac{x}{2} = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$ (30°)

Since we will be doubling final solutions, there is no need to add multiples of 2π to primary solutions. (Otherwise, doubling will produce values greater than 4π, which are not primary solutions.)

$\frac{x}{2}$ in quadrant I: $\frac{x}{2} = \frac{\pi}{6}$ (30°), so $x = \frac{\pi}{3}$ (60°);

$\frac{x}{2}$ in quadrant IV: $\frac{x}{2} = \frac{11\pi}{6}$ (330°), so $x = \frac{11\pi}{3}$ (660°).

Discarding the solution greater than 2π, we obtain $x = \frac{\pi}{3}$ (60°).

79. $\sin^2\frac{x}{4} = \frac{1}{2}$

$\sin\frac{x}{4} = \pm\frac{\sqrt{2}}{2}$

$\left(\frac{x}{4}\right)' = \sin^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$ (45°)

$\frac{x}{4} = \frac{\pi}{4}$ (45°), $\frac{3\pi}{4}$ (135°), $\frac{5\pi}{4}$ (225°), $\frac{7\pi}{4}$ (315°), so, multiplying by 4 and discarding non-primary solutions,

$x = \pi$ (180°).

81. $3\tan^2\frac{\theta}{4} = 9$

$\tan^2\frac{\theta}{4} = 3$

$\tan\frac{\theta}{4} = \pm\sqrt{3}$

$\left(\frac{\theta}{4}\right)' = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$ (60°)

$\frac{\theta}{4} = \frac{\pi}{3}$ (60°), $\frac{2\pi}{3}$ (120°), $\frac{4\pi}{3}$ (240°), $\frac{5\pi}{3}$ (300°). Multiply by 4, discard values greater than 2π (360°): $\theta = \frac{4\pi}{3}$ (240°)

83. $\cos\theta + \sin 2\theta = 0$

$\cos\theta + 2\sin\theta\cos\theta = 0$

$\cos\theta(1 + 2\sin\theta) = 0$

$\cos\theta = 0$

$\frac{\pi}{2}$ (90°), $\frac{3\pi}{2}$ (270°)

$\sin\theta = -\frac{1}{2}$

$\frac{7\pi}{6}$ (210°), $\frac{11\pi}{6}$, (330°)

85. $(\cot 4x - \sqrt{3})(\csc 3x + 2) = 0$

$\cot 4x - \sqrt{3} = 0$

$\cot 4x = \sqrt{3}$

$\tan 4x = \frac{\sqrt{3}}{3}$

$(4x)' = \frac{\pi}{6}, \frac{7\pi}{6}$

$4x = \frac{\pi}{6} + k\pi$

$\csc 3x + 2 = 0$

$\csc 3x = -2$

$\sin 3x = -\frac{1}{2}$

$(3x') = \frac{7\pi}{6}, \frac{11\pi}{6}$

$3x = \frac{7\pi}{6} + 2k\pi,\ \frac{11\pi}{6} + 2k\pi$

$x = \frac{\pi}{24} + k\bullet\frac{\pi}{4}$ or $\frac{7\pi}{18} + k\bullet\frac{2\pi}{3}$ or $\frac{11\pi}{18} + k\bullet\frac{2\pi}{3}$

$x \approx 0.13 + k\bullet\frac{\pi}{4}$ or $1.22 + k\bullet\frac{\pi}{4}$ or $1.92 + k\bullet\frac{\pi}{4}$

1. $\csc^2 x \sin x \cos x$

$\dfrac{1}{\sin^2 x} \sin x \cos x$

$\dfrac{1}{\sin x} \cos x$

$\dfrac{\cos x}{\sin x}$

$\cot x$

3. $\cot x - 2 \tan x$

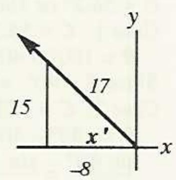
$\cot \frac{\pi}{4} - 2 \tan \frac{\pi}{4}$

$1 - 2(1)$

$-1 \neq 1$

5. $\sin 2x = 2 \sin x \cos x$

$= 2\,\frac{15}{17}\left(-\frac{8}{17}\right) = -\frac{240}{289}$



7. $\cos 22.5° = \cos \frac{45°}{2} = \sqrt{\dfrac{1 + \cos 45°}{2}} = \sqrt{\dfrac{1 + \frac{\sqrt{2}}{2}}{2}}$

$= \sqrt{\dfrac{2 + \sqrt{2}}{4}} = \dfrac{\sqrt{2 + \sqrt{2}}}{2}$. $\sec 22.5° = \dfrac{1}{\cos 22.5°}$

$= \dfrac{2}{\sqrt{2 + \sqrt{2}}} \cdot \dfrac{\sqrt{2 - \sqrt{2}}}{\sqrt{2 - \sqrt{2}}} = \dfrac{2\sqrt{2 - \sqrt{2}}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{2(2 - \sqrt{2})}}{2}$

$= \sqrt{4 - 2\sqrt{2}}$. (Note: $\dfrac{2}{\sqrt{2 + \sqrt{2}}}$ given in the text)

9. $4 \cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm \frac{1}{2}$

$x' = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}\,(60°)$

$x = x', \pi - x', \pi + x', 2\pi - x'$

$= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

11. $(6 \sin x - 1)(\sin x + 1) = 0$

$6 \sin x - 1 = 0$

$\sin x = \frac{1}{6}$

$x' = \sin^{-1} \frac{1}{6} \approx 0.167$

$x = x'$ or $\pi - x'$

$= 0.17$ or 2.97

$\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

The solutions are $x \approx 0.17 + 2k\pi$, $2.97 + 2k\pi$, or $\frac{3\pi}{2} + 2k\pi$.

13. $\sec^2 \frac{x}{4} = 2$

$\cos^2 \frac{x}{4} = \frac{1}{2}$

$\cos \frac{x}{4} = \pm \frac{\sqrt{2}}{2}$

$\left(\frac{x}{4}\right)' = \frac{\pi}{4}$

Primary solutions for $\frac{x}{4}$ are $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

$\frac{x}{4}$	x (all solutions)
$\frac{\pi}{4} + 2k\pi$	$\pi + 8k\pi$
$\frac{3\pi}{4} + 2k\pi$	$3\pi + 8k\pi$
$\frac{5\pi}{4} + 2k\pi$	$5\pi + 8k\pi$
$\frac{7\pi}{4} + 2k\pi$	$7\pi + 8k\pi$

Exercise 6-1

1. $a = 12.5, A = 35°, B = 49°$
$C = 180° - A - B = 96°$
$$\frac{\sin 35°}{12.5} = \frac{\sin 49°}{b} = \frac{\sin 96°}{c}$$
$\frac{\sin 35°}{12.5} = \frac{\sin 49°}{b}$ $\qquad$ $\frac{\sin 35°}{12.5} = \frac{\sin 96°}{c}$
$b = \frac{12.5 \sin 49°}{\sin 35°}$ $\qquad$ $c = \frac{12.5 \sin 96°}{\sin 35°}$
$b \approx 16.4$ $\qquad$ $c \approx 21.7$
Keystrokes for b:
12.5 $\boxed{\times}$ 49 $\boxed{\sin}$ $\boxed{\div}$ 35 $\boxed{\sin}$ $\boxed{=}$
TI-81: 12.5 $\boxed{\sin}$ 49 $\boxed{\div}$ $\boxed{\sin}$ 35 $\boxed{\text{ENTER}}$

5. $b = 92.5, A = 47°, B = 100°$
$C = 180° - 47° - 100° = 33°$
$$\frac{\sin 47°}{a} = \frac{\sin 100°}{92.5} = \frac{\sin 33°}{c}$$
$\frac{\sin 100°}{92.5} = \frac{\sin 33°}{c}$ $\qquad$ $\frac{\sin 47°}{a} = \frac{\sin 100°}{92.5}$
$c = \frac{92.5 \sin 33°}{\sin 100°}$ $\qquad$ $a = \frac{92.5 \sin 47°}{\sin 100°}$
$c \approx 51.2°$ $\qquad$ $a \approx 68.7$

9. $c = 5.00, A = 100°, B = 45°$
$C = 180° - 100° - 45° = 35°$
$$\frac{\sin 100°}{a} = \frac{\sin 45°}{b} = \frac{\sin 35°}{5}$$
$\frac{\sin 100°}{a} = \frac{\sin 35°}{5}$ $\qquad$ $\frac{\sin 45°}{b} = \frac{\sin 35°}{5}$
$a = \frac{5 \sin 100°}{\sin 35°}$ $\qquad$ $b = \frac{5 \sin 45°}{\sin 35°}$
$a \approx 8.58$ $\qquad$ $b \approx 6.16$

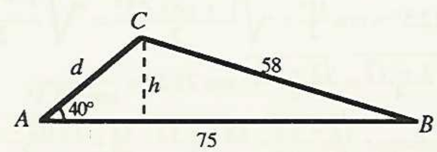
13. $a = 4.25, c = 2.86, A = 132°$
$$\frac{\sin 132°}{4.25} = \frac{\sin B}{b} = \frac{\sin C}{2.86}$$
$\frac{\sin 132°}{4.25} = \frac{\sin C}{2.86}$ so $\sin C = \frac{2.86 \sin 132°}{4.25}$,
$C' = \sin^{-1} \frac{2.86 \sin 132°}{4.25} \approx 30.01°$
$C \approx 30.01°$ or $180° - 30.01° \approx 149.99°$
Case 1: $C \approx 30.01°$
$B \approx 180° - 132° - 30.01° \approx 17.99°$
$\frac{\sin 132°}{4.25} \approx \frac{\sin 17.99°}{b}$
$b \approx 1.77$
Thus, the solution is $B \approx 18.0°, C \approx 30.0°, b \approx 1.77$.
Case 2: $C \approx 149.99°$
$B = 180° - 132° - 149.99° \approx -101.99$ (No solution.)

17. $a = 4, b = 22, A = 30°$
$$\frac{\sin 30°}{4} = \frac{\sin B}{22} = \frac{\sin C}{c}$$
$\frac{\sin 30°}{4} = \frac{\sin B}{22}$ so $\sin B = \frac{22 \sin 30°}{4}$,
$B' = \sin^{-1} \frac{22 \sin 30°}{4} = \sin^{-1} 2.75$
B' is undefined, since $0 < \sin B < 1$, and we have $\sin B = 2.75$ from this data.
Thus, there is no solution possible.

21. $c = 5.00, b = 8.00, B = 45.0°$
$$\frac{\sin A}{a} = \frac{\sin 45°}{8} = \frac{\sin C}{5}$$
$\frac{\sin 45°}{8} = \frac{\sin C}{5}$, so $\sin C = \frac{5 \sin 45°}{8}$
$C' = \sin^{-1} \frac{5 \sin 45°}{8} \approx 26.23°$.
$C \approx 26.23°$ or $180° - 26.23° \approx 153.77°$
Case 1: $C \approx 26.23°$
$A \approx 180° - 45° - 26.23° \approx 108.77°$
$\frac{\sin 108.77°}{a} = \frac{\sin 45°}{8}$
$a \approx 10.71$
Solution : $\quad C \approx 26.2°, A \approx 108.77°, a \approx 10.71$
Case 2: $C \approx 153.77°$
$A \approx 180° - 45° - 153.77° \approx -18.77°$
No solution.

25. The acute angle at the asteroid is $180° - 81.5° - 88° = 10.5°$.
$\frac{\sin 10.5°}{153\ 800} = \frac{\sin 88°}{d}$; d ≈ 843449 ≈ 843,400 miles.

29. $\frac{\sin 40°}{58} = \frac{\sin C}{75}$; $\sin C = \frac{75 \sin 40°}{58}$; $C' \approx 56.2°$.
$C \approx 56.2°$ or $180° - 56.2° \approx 123.8°$.
Case 1: $C \approx 56.2°$
$B = 180° - 40° - 56.2° \approx 83.8°$
Since $B < 40°$, we solve the second case.
Case 2: $C \approx 123.8°$
$B = 180° - 40° - 123.8° \approx 16.2°$
$\frac{\sin 40°}{58} = \frac{\sin 16.2°}{d}$; $d \approx 25.17$
Thus $d \approx 25$ miles.



33. Let (x, y) be the point at B. It is on the terminal side of angle A. Then $\cos A = \frac{x}{r}$, where r is the length of AB. But $r = c$, so $\cos A = \frac{x}{c}$ and so $c \cos A = x$.
Using right triangles we see that in each figure $\cos C = \frac{b - x}{a}$ so that $a \cos C = b - x$. Note that when A is obtuse (the right hand figure) x is negative, so $b - x$ is the length of $|b| + |x|$.
$$\begin{aligned} c \cos A &= x \\ \underline{a \cos C} &= \underline{b - x} \\ a \cos C + c \cos A &= x + (b - x) \qquad \text{Add} \\ a \cos C + c \cos A &= b \end{aligned}$$
Thus (2) is true.
(1) and (3) can be shown true by putting angles B and C in standard position and proceding in the same manner. In fact this is not really necessary, since the labelling in a triangle is arbitrary, and thus, for example, we could obtain (1) by changing the label B to A, C to B, and A to C, and labelling the sides appropriately. (Also see the discussion in the text).

37. (a) It can be seen that the sum of the area of the four triangles shown in the figure is
$\frac{1}{2} ab \sin A + \frac{1}{2} cd \sin C + \frac{1}{2} ad \sin D + \frac{1}{2} bc \sin B$.
This total is twice as large as the total area of the four–sided figure, so the area of the four–sided figure is
$\frac{1}{2}\left(\frac{1}{2} ab \sin A + \frac{1}{2} cd \sin C + \frac{1}{2} ad \sin D + \frac{1}{2} bc \sin B\right)$
or $\frac{1}{4}(ab \sin A + ad \sin D + bc \sin B + cd \sin C)$.
(b) The difference between the Egyptian formula $\frac{1}{4}(ab + ad + bc + cd)$ and the correct formula $\frac{1}{4}(ab \sin A + ad \sin D + bc \sin B + cd \sin C)$ is the factors sin A, sin B, sin C, and sin D. Since we assumed each angle is between 0° and 180° the value of the sine of each angle is

between 0 and 1.

Thus, $ab \geq ab \sin A$

$ad \geq ad \sin D$

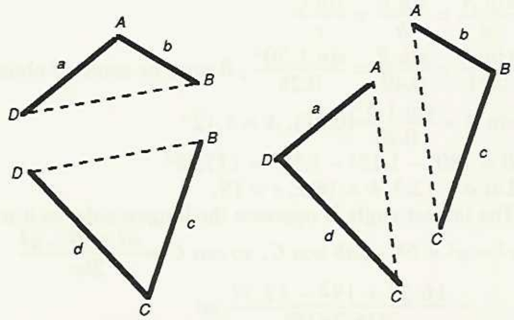
$bc \geq bc \sin B$

$cd \geq cd \sin C$

$ab + ad + bc + cd \geq ab \sin A + ad \sin D + bc \sin B + cd \sin C,$

$\frac{1}{4}(ab + ad + bc + cd) \geq \frac{1}{2}(ab \sin A + ad \sin D + bc \sin B + cd \sin C).$

If the figure is a rectangle $A = B = C = D = 90°$, and $\sin A = \sin B = \sin C = \sin D = 1$, so both expressions give the same value.



Exercise 6–2

1. $(3,-4), (-2,6)$: $d = \sqrt{(-2-3)^2 + (6-(-4))^2} = \sqrt{125} = 5\sqrt{5}$

5. $(0,-3), (-9,-12)$: $d = \sqrt{(-9-0)^2 + (-12-(-3))^2} = 9\sqrt{2}$

9. $b = 61.3,\ c = 23.9,\ A = 124.0°$

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 61.3^2 + 23.9^2 - 2(61.3)(23.9) \cos 124°$

$a = \sqrt{61.3^2 + 23.9^2 - 2(61.3)(23.9) \cos 124°} \approx 77.249$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 124°}{77.249} = \frac{\sin B}{61.3} = \frac{\sin C}{23.9}$ Find angle C first; it is the smallest and therefore acute.

$\sin C \approx \frac{23.9 \sin 124°}{77.249}$; $C \approx 14.9°$

$B \approx 180° - 14.9° - 124° \approx 41.1°$

Thus $a \approx 77.2$, $B \approx 41.1°$, $C \approx 14.9°$.

13. $a = 23.5,\ b = 19.4,\ c = 35.0$

$c^2 = a^2 + b^2 - 2ab \cos C$, so $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{23.5^2 + 19.4^2 - 35^2}{2(23.5)(19.4)}$ so $C \approx 108.97° \approx 109.0°$

23.5 $\boxed{x^2}$ $\boxed{+}$ 19.4 $\boxed{x^2}$ $\boxed{-}$ 35 $\boxed{x^2}$ $\boxed{=}$ $\boxed{\div}$ 2 $\boxed{\div}$ 23.5 $\boxed{\div}$ 19.4 $\boxed{=}$ $\boxed{\text{SHIFT}}$ $\boxed{\text{cos}}$

$\boxed{\text{TI-81}}$ $\boxed{\text{2nd}}$ $\boxed{\text{COS}}$ $\boxed{(}$ 23.5 $\boxed{x^2}$ $\boxed{+}$ 19.4 $\boxed{x^2}$ $\boxed{-}$ 35 $\boxed{x^2}$ $\boxed{)}$ $\boxed{\div}$ 2 $\boxed{\div}$ 23.5 $\boxed{\div}$ 19.4 $\boxed{\text{ENTER}}$

Since C is the largest angle in the triangle we know A and B are acute.

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin A}{23.5} = \frac{\sin B}{19.4} = \frac{\sin 108.97°}{35}$

$\sin A = \frac{\sin 108.97°}{35}(23.5)$; $A \approx 39.4°$; $B \approx 180° - 109.0° - 39.4° \approx 31.6°$

17. $a = 13.2,\ b = 5.9,\ C = 139.4°$

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 13.2^2 + 5.9^2 - 2(13.2)(5.9) \cos 139.4°$

$c = \sqrt{13.2^2 + 5.9^2 - 2(13.2)(5.9) \cos 139.4°} \approx 18.09 \approx 18.1$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin A}{13.2} = \frac{\sin B}{5.9} = \frac{\sin 139.4°}{18.09}$; $\sin A \approx \frac{\sin 139.4°}{18.09}(13.2)$, so $A \approx 28.3°$ (A is acute.)

$B \approx 180° - 139.4° - 28.3° \approx 12.3°$.

21. $a = 30.0,\ c = 20.0,\ B = 112.0°$

$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 30^2 + 20^2 - 2(30)(20) \cos 112°$

$b \approx 41.83 \approx 41.8$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin A}{30} = \frac{\sin 112°}{41.8} = \frac{\sin C}{20}$; A and C must both be acute.

$\sin C \approx \frac{\sin 112°}{41.8}(20)$, so $C \approx 26.3°$

$A \approx 180° - 26.3° - 112° \approx 41.7°$

25. $a = 0.21,\ b = 0.49,\ C = 1.50°$

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 0.21^2 + 0.49^2 - 2(0.21)(0.49) \cos 1.50°$, so $c \approx 0.280$

≈ 0.28

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\dfrac{\sin A}{0.21} = \dfrac{\sin B}{0.49} = \dfrac{\sin 1.50°}{0.28}$; B may be acute or obtuse, so find A first.

$\sin A \approx \dfrac{\sin 1.5°}{0.28}(0.21)$, $A \approx 1.12°$

$B \approx 180° - 1.12° - 1.50° \approx 177.38°$.

29. Let $a = 12.3$, $b = 16.2$, $c = 19$.
The largest angle is opposite the longest side, so it is angle C.

$c^2 = a^2 + b^2 - 2ab \cos C$, so $\cos C = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{16.2^2 + 19^2 - 12.3^2}{2(16.2)(19)}$, so

$C \approx 82.4°$, to the nearest tenth.

33. $31.5^2 = 17.6^2 + 22.5^2 - 2(17.6)(22.5) \cos \theta°$

$\cos \theta = \dfrac{17.6^2 + 22.5^2 - 31.5^2}{2(17.6)(22.5)}$; $\theta \approx 102.9°$

37. Angle A is the largest. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{53 + 34 - 65}{2\sqrt{53}(\sqrt{34})} \approx 0.2591$, $A \approx 75.0°$.

Exercise 6–3

We use $A = (|A|, \theta_A) = (|A| \cos \theta_A, |A| \sin \theta_A)$.

1. $(40, 30°) = (40 \cos 30°, 40 \sin 30°) = (40 \cdot \frac{\sqrt{3}}{2}, 40 \cdot \frac{1}{2}) =$
$(20\sqrt{3}, 20)$.

5. $(10.0, 200.0°) = (10 \cos 200°, 10 \sin 200°) \approx (-9.4, -3.4)$

9. $(6, -45°) = (6 \cos(-45°), 6 \sin(-45°)) \approx$
$(6 \cos 45°, -6 \sin 45°)$ cosine is an even function, sine is an odd function.
$= (6 \cdot \frac{\sqrt{2}}{2}, -6 \cdot \frac{\sqrt{2}}{2}) = (3\sqrt{2}, -3\sqrt{2})$.

13. (3.0. 4.0): $|A| = \sqrt{3^2 + 4^2} = 5$, $\theta' = \tan^{-1} \frac{4}{3} \approx 53.13°$
(5, 53.1°)

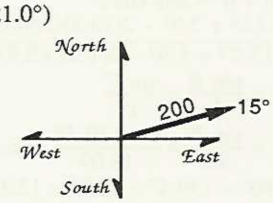
17. $(\sqrt{3}, -2)$; $|A| = \sqrt{(\sqrt{3})^2 + (-2)^2} \approx 2.646$, $\theta' = \tan^{-1} \frac{-2}{\sqrt{3}} \approx$
$-49.11°$; $V_x > 0$ so $\theta = \theta'$: (2.6, –49.1°)

21. (–6.8, 3.4); $|A| = \sqrt{(-6.8)^2 + 3.4^2} \approx 7.60$, $\theta' \approx -26.57°$; $\theta \approx$
$-26.57° + 180° \approx 153.43°$; (7.6, 153.4°)

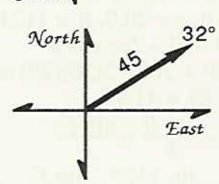
25. (–3, 8), (2, 12) (–3 + 2, 8 + 12)= (–1, 20)

29.	(30.0, 30°)	= (30 cos 30°, 30 sin 30°)	= (25.980, 15.000)	
	(15.2, 33.6°)	= (15.2 cos 33.6°, 15.2 sin 33.6°)	= (12.660, 8.412)	
			(38.641, 23.412)	= (45.2, 31.2°)
33.	(3.2, –45.0°)	= (3.2 cos(–45°), 3.2 sin(–45°)	= (2.263, –2.263)	
	(5.9, –59.2°)	= (5.9 cos(–59.2°), 5.9 sin(–59.2°))	= (3.021, –5.068)	
			(5.284, –7.331)	= (9.0, –54.2°)
37.	(3.5, 19.2°)	= (3.5 cos 19.2°, 3.5 sin 19.2°)	= (3.305, 1.151)	
	(2.7, 83.1°)	= (2.7 cos 83.1°, 2.7 sin 83.1°)	= (0.324, 2.680)	
	(4.3, 145.7°)	= (4.3 cos 145.7°, 4.3 sin 145.7°)	= (–3.552, 2.423)	
			= (0.077, 6.255)	= (6.3, 89.3°)
41.	(3.5, –25°)	= (3.5 cos(–25°), 3.5 sin(–25°))	= (3.172, –1.479)	
	(6.8, 25°)	= (6.8 cos 25°, 6.8 sin 25°)	= (6.163, 2.874)	
	(4.2, 50°)	= (4.2 cos 50°, 4.2 sin 50°)	= (2.700, 3.217)	
			= (12.035, 4.612)	= (12.9, 21.0°)

45. $V = (200, 15°)$; $V_x = 200 \cos 15° \approx 193$; $V_y = 200 \sin 15° \approx 52$.
The aircraft is flying east at 193 knots and north at 52 knots.



49. 18 knots (18 nautical miles per hour) x 2.5 hours = 45 nm (nautical miles).
$V = (45, 32°)$; $V_x = 45 \cos 32° \approx 38$ nm; distance east of the harbor (part b)
$V_y = 45 \sin 32° \approx 24$ nm; distance north of the harbor (part a).

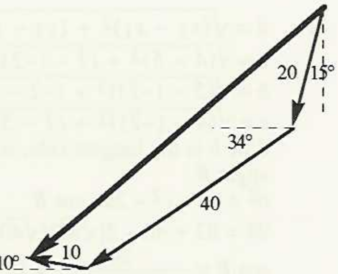


53.

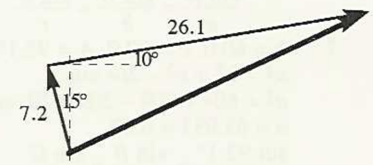
Force V	Horizontal Component V_x	Vertical Component V_y	
(1000, 15°)	966	259	Part (a).
(2000, 15°)	1932	518	Part (b); yes, the components double in value.
(1000, 30°)	866	500	Part (c); no, the components do not double.

57.	(2.6, 18.3°)	= (2.6 cos 18.3°, 2.6 sin 18.3°)	≈ (2.47, 0.82)	
	(15.8, –86.2°)	= (15.8 cos(–86.2°, 15.8 sin(–86.2°)	≈ (1.05, –15.77)	
			≈ (3.52, –14.95)	≈ (15.4, –76.8°)

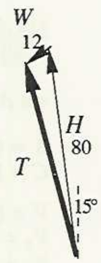
61. $(20, 255°) + (40, 214°) + (10, 170°) \approx (62.6, -140.3°)$. Thus the ship is about 63 nautical miles from its starting position, at an angle of 40° south of west.



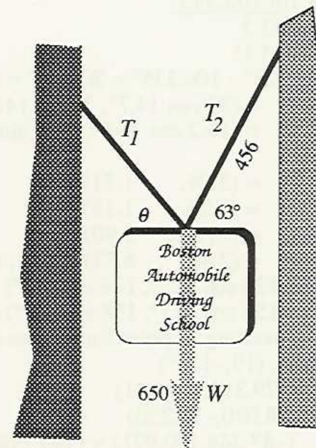
65. $(26.1, 10°) + (7.2, 105°) \approx (26.5, 25.7°)$ Thus the ship is traveling at 26.5 knots in a direction 25.7° north of east.



69. $H + W = T$, so $H = T - W$
$= (80, 105°) - (12, 225°)$
$= (80, 105°) + (12, 225° - 180°)$
$= (80, 105°) + (12, 45°)$
$= (-20.71, 77.27) + (8.49, 8.49)$
$= (-12.22, 85.76) \approx (87, 98°)$.
Thus the heading of the aircraft is 8° west of north, and its airspeed is 87 knots.



73. The sign is stationary, so the forces acting on it are balanced (they add to zero).
$T_1 + T_2 + W = 0$
$T_1 = -T_2 - W$
$= -(456, 63°) - (650, 270°)$
$= (456, 63° + 180°) + (650, 270° - 180°)$ To negate a vector, add or subtract 180° from its direction angle.
$= (456, 243°) + (650, 90°)$
$\approx (-207.02, -406.3) + (0, 650)$ Convert to rectangular form.
$\approx (-207.02, 243.7)$
$\approx (320, 130°)$ Convert back to polar form.
Thus the tension in the second cable is 320 pounds, and it makes an angle θ of 50° (180° − 130°) with the horizontal.

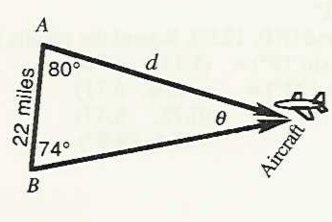


Chapter 6 Review

1. $a = 10.6, A = 47.9°, B = 10.3°$
$C = 180° - 47.9° - 10.3° = 121.8°$
$$\frac{\sin 47.9°}{10.6} = \frac{\sin 10.3°}{b} = \frac{\sin 121.8°}{c}$$
$$b = \frac{10.6 \sin 10.3°}{\sin 47.9°} \approx 2.6$$
$$c = \frac{10.6 \sin 121.8°}{\sin 47.9°} \approx 12.1$$

3. $a = 10.0, b = 13.0, B = 79.0°$
$$\frac{\sin A}{10} = \frac{\sin 79°}{13} = \frac{\sin C}{c}$$
Observe that A must be acute, since side a is not the longest side of the triangle.
$$\sin A = \frac{10 \sin 79°}{13}, \text{ so } A \approx 49.03° \approx 49.0°$$
$C \approx 180° - 79° - 49.03° \approx 51.97° \approx 52.0°$.
$$\frac{\sin 79°}{13} \approx \frac{\sin 51.97°}{c}, \text{ so } c \approx \frac{13 \sin 51.97°}{\sin 79°} \approx 10.4$$

5. $\theta = 180° - 80° - 74° = 26°$; $\dfrac{\sin 26°}{22} = \dfrac{\sin 74°}{d}$
$$d = \frac{22 \sin 74°}{\sin 26°};\ d \approx 48 \text{ miles}$$

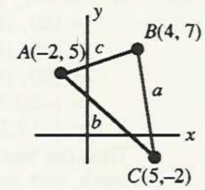


$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

7. $b = 60.0$, $c = 20.0$, $A = 92.1°$
$a^2 = b^2 + c^2 - 2bc \cos A$
$a^2 = 60^2 + 20^2 - 2(60)(20) \cos 92.1°$
$a \approx 63.937 \approx 63.9$
$\frac{\sin 92.1°}{63.937} = \frac{\sin B}{60} = \frac{\sin C}{20}$
Neither B nor C can be obtuse, since angle A must be the largest angle in the triangle.
$\sin B = \frac{60 \sin 92.1°}{63.937}$, so $B \approx 69.7°$
$C \approx 180° - 69.7° - 92.1° \approx 18.2°$

9. $a = 43.5$, $b = 17.8$, $c = 35.0$
Find the largest angle first, since the two smallest angles must be acute, and the law of cosines does not have an ambiguous case. The largest angle is opposite the longest side, so it is A.
$a^2 = b^2 + c^2 - 2bc \cos A$
$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{17.8^2 + 35^2 - 43.5^2}{2(17.8)(35)}$
$A \approx 106.335° \approx 106.3°$
$\frac{\sin 106.335°}{43.5} = \frac{\sin B}{17.8} = \frac{\sin C}{35}$
$\sin B = \frac{17.8 \sin 106.335°}{53.5}$
$B \approx 23.123° \approx 23.1°$
$C \approx 180° - 23.123° - 106.335° \approx 50.542° \approx 50.5°$

11. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$a = \sqrt{(4-5)^2 + (7-(-2))^2} = \sqrt{82}$
$b = \sqrt{(5-(-2))^2 + (-2-5)^2} = \sqrt{98} = 7\sqrt{2}$
$c = \sqrt{(4-(-2))^2 + (7-5)^2} = \sqrt{40} = 2\sqrt{10}$
Side b is the longest side, so we use the law of cosines to find angle B.
$b^2 = a^2 + c^2 - 2ac \cos B$
$98 = 82 + 40 - 2(\sqrt{82})(\sqrt{40}) \cos B$
$\cos B = \frac{24}{2(\sqrt{82})(\sqrt{40})} \approx 0.20953$, so $B \approx 77.905° \approx 77.9°$
$\frac{\sin A}{\sqrt{82}} = \frac{\sin 77.905°}{\sqrt{98}}$
$\sin A = \frac{\sqrt{82}}{\sqrt{98}} \sin 77.905°$
$A \approx 63.4°$
$C \approx 180° - 77.9° - 63.4° \approx 38.7°$



13. $V = (27.2, 29°)$
$V_x = 27.2 \cos 29° \approx 23.8$
$V_y = 27.2 \sin 29° \approx 13.2$

15. Convert (450,34.6°) to rectangular coordinates.
$(450, 34.6°) = (450 \cos 34.6°, 450 \sin 34.6°) \approx (370, 256)$
Thus the horizontal component is 370 knots, and the vertical component is 256 knots.

17. $(33.0, 14.7°) = (33 \cos 14.7°, 33 \sin 14.7°) \approx (31.92, 8.37)$
$(15.2, 33.6°) = (15.2 \cos 33.6°, 15.2 \sin 33.6°) \approx (12.66, 8.41)$
$(44.58, 16.79) \approx (47.6, 20.6°)$

19. $(3.5, 29.2°) \approx (3.06, 1.71)$
$(1.7, 43.1°) \approx (1.24, 1.16)$
$(4.3, 115.0°) \approx (-1.82, 3.90)$
$\approx (2.48, 6.77) \approx (7.2, 69.9°)$

21. $(126, 223°) = (126 \cos 223°, 126 \sin 223°) \approx (-92.15, -85.93)$
$(158, 311°) = (158 \cos 311°, 158 \sin 311°) \approx (103.66, -119.24)$
Adding and converting to polar form gives magnitude ≈ 205 pounds; direction ≈ −87°.

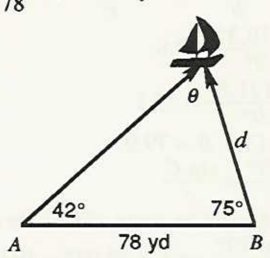
23. Add (195,114°), (19,−115°)
$(195,114°) \approx (-79.314, 178.141)$
$(19,-115°) \approx (-8.030, -17.220)$
$(-87.344, 160.921) \approx (183, 118°)$
Ground speed is 183 knots, true course is 62° north of west.

Chapter 6 Test

1. $B = 180° - 13.5° - 82.1° = 84.4°$
$\frac{\sin 13.5°}{a} = \frac{\sin 84.4°}{22.6} = \frac{\sin 82.1°}{c}$
$a = \frac{22.6 \sin 13.5°}{\sin 84.4°} \approx 5.3$
$c = \frac{22.6 \sin 82.1°}{\sin 84.4°} \approx 22.5$

3. $b^2 = a^2 + c^2 - 2ac \cos B$
$b^2 = 25.9^2 + 16.2^2 - 2(25.9)(16.2) \cos 100° \approx 1078.97$
$b \approx 32.848 \approx 32.8$
$\frac{\sin A}{a} = \frac{\sin B}{b}$
$\frac{\sin A}{25.9} = \frac{\sin 100°}{32.848}$
$\sin A = \frac{25.9 \sin 100°}{32.848}$; $A \approx 50.9°$
$C \approx 180° - 50.9° - 100° \approx 29.1°$

5. $\theta = 180° - 42° - 75° = 63°$
$\frac{\sin 42°}{d} = \frac{\sin 63°}{78}$; $d \approx 59$ yards

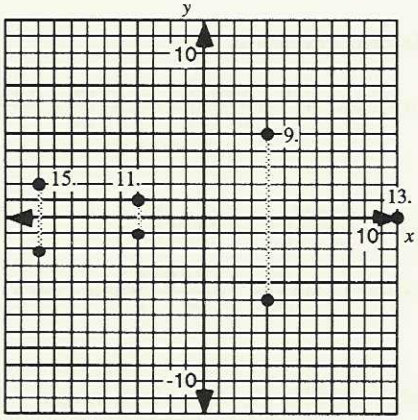


7. $(2, 30°) = (2 \cos 30°, 2 \sin 30°)$
$= \left(2 \cdot \frac{\sqrt{3}}{2}, 2 \cdot \frac{1}{2}\right) = (\sqrt{3}, 1)$

9. Add the vectors (5.4, 19.0°) and (8.0, 123°). Round the results to the nearest tenth.
$(5.4, 19°) = (5.4 \cos 19°, 5.4 \sin 19°) \approx (5.11, 1.76)$
$(8, 123°) = (8 \cos 123°, 8 \sin 123°) \approx (-4.36, 6.71)$
$(0.75, 8.47)$
$\approx (8.5, 84.9°)$

Chapter 7

Exercise 7-1



1. real: 4; im: –5
5. real: 12; im: 0
9. $4 + 5i$
11. $-4 - i$
13. 12
15. $-10 - 2i$

17. $(5 + 4i) + (-3 + 2i) = 2 + 6i$
21. $13 - 7i + 3i - 4 = 9 - 4i$
25. $-5i(6 - 4i) = -30i + 20i^2 = -20 - 30i$
29. $\dfrac{5 + 4i}{5 + 2i} \cdot \dfrac{5 - 2i}{5 - 2i} = \dfrac{33 + 10i}{29} = \dfrac{33}{29} + \dfrac{10}{29}i$

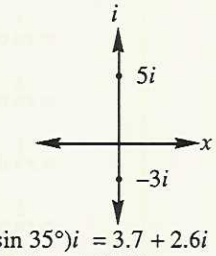
> Remember: Given vector $z = a + bi = r \text{ cis } \theta$. Then
> $$r = \sqrt{a^2 + b^2}, \qquad \tan\theta = \tan^{-1}\frac{b}{a}, \text{ and}$$
> $$\theta = \begin{cases} \theta' & \text{if } a > 0 \\ \theta' - 180° \text{ if } \theta' > 0 & \text{if } a < 0 \\ \theta' + 180° \text{ if } \theta' < 0 & \end{cases}.$$

33. $5 - 2i$
$a = 5, b = -2;\ r = \sqrt{a^2 + b^2} = \sqrt{5^2 + (-2)^2} = \sqrt{29} \approx 5.4$.
$\theta' = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{-2}{5} \approx -21.8°$.
Since $a > 0$ $\theta = \theta'$. Thus the point is 5.4 cis(–21.8°).

37. $-3 + 4i$ $\quad r = \sqrt{(-3)^2 + 4^2} = 5$
$\theta' = \tan^{-1}\frac{4}{-3} \approx -53.1°$. $a < 0$, $\theta' < 0$ so $\theta \approx -53.1° + 180° \approx 126.9°$. The point is 5 cis 126.9°.

41. $3 + 3i$ $\quad r = \sqrt{3^2 + 3^2} = 3\sqrt{2}$.
$\theta' = \tan^{-1} 1 = 45°$; the point is $3\sqrt{2}$ cis 45°.

45. $5i$ Plotting the point shows that $r = 5$ and $\theta = 90°$. Thus the point is 5 cis 90°.



49. 4.5 cis 35° $\quad 4.5 \cos 35° + (4.5 \sin 35°)i = 3.7 + 2.6i$
53. 13.6 cis (–25°) $\quad 13.6 \cos(-25°) + (13.6 \sin(-25°))i = 12.3 - 5.7i$
57. 10 cis 300° $\quad 10 \cos 300° + (10 \sin 300°)i = 10 \cdot \frac{1}{2} + 10(-\frac{\sqrt{3}}{2})\,i = 5 - 5\sqrt{3}\,i$
61. $\sqrt{8}$ cis 315° $\quad \sqrt{8} \cos 315° + (\sqrt{8} \sin 315°)i = \sqrt{8} \cdot \frac{\sqrt{2}}{2} + \sqrt{8} \cdot (-\frac{\sqrt{2}}{2})i = 2 - 2i$
65. (5.4 cis 300°)(2 cis 300°) = 10.8 cis(600°)
= 10.8 cis(600° – 2·360°) = 10.8 cis(–120°)°
69. $\dfrac{40 \text{ cis } 80°}{18 \text{ cis } 160°} = \dfrac{40}{18}\text{cis}(80° - 160°) = \dfrac{20}{9}\text{ cis}(-80°)$
73. $(3 \text{ cis } 200°)^3 = 3^3 \text{ cis}(3 \cdot 200°) = 27 \text{ cis } 600°$
$= 27 \text{ cis}(600° - 2 \cdot 360°) = 27 \text{ cis}(-120°)$

> Use: $(r \text{ cis } \theta)^{\frac{1}{n}} = r^{\frac{1}{n}} \text{cis}\left(\dfrac{\theta}{n} + \dfrac{k \cdot 360°}{n}\right)$, $0 \le k < n$.

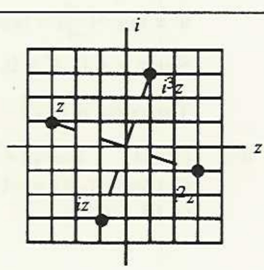
77. Find the 3 cube roots of 8 in exact form.
8 = 8 cis 0°
Evaluate $8^{\frac{1}{3}} \text{ cis}\left(\dfrac{0°}{3} + \dfrac{k \cdot 360°}{3}\right) = 2 \text{ cis } (k \cdot 120°)$ for $k = 0, 1, 2$.
$k = 0$: $2 \text{ cis } 0° = 2 \cos 0° + 2 \sin 0° i = 2 + 0i = 2$
$k = 1$: $2 \text{ cis } 120° = 2 \cos 120° + 2 \sin 120° i$
$= 2\left(-\frac{1}{2}\right) + 2\left(\frac{\sqrt{3}}{2}\right)i = -1 + \sqrt{3}i$
$k = 2$: $2 \text{ cis } 240° = 2 \cos 240° + 2 \sin 240° i$
$= 2\left(-\frac{1}{2}\right) + 2\left(-\frac{\sqrt{3}}{2}\right)i = -1 - \sqrt{3}i$
Thus the three cube roots of 8 are $2, -1 + \sqrt{3}\,i, -1 - \sqrt{3}\,i$.

81. Find the 3 cube roots of $75 - 100i$ to the nearest tenth.
$75 - 100i \approx 125 \text{ cis}(306.8699°)$; $(75 - 100i)^{1/3} \approx (125 \text{ cis}(306.8699°))^{1/3}$
Evaluate $\sqrt[3]{125} \text{ cis}\left(\dfrac{306.8699°}{3} + \dfrac{k \cdot 360°}{3}\right) = 5 \text{ cis}(102.29° + k \cdot 120°)$ for $k = 0, 1, 2$.
$k = 0$: $5 \text{ cis}(102.29°) = 5 \cos 102.29° + 5 \sin 102.29°\, i \approx -1.1 + 4.9i$
$k = 1$: $5 \text{ cis}(102.29° + 120°) = 5 \cos 222.29° + 5 \sin 222.29°\, i \approx -3.7 - 3.4i$
$k = 2$: $5 \text{ cis}(102.29° + 240°) = 5 \cos 342.29° + 5 \sin 342.29°\, i \approx 4.8 - 1.5i$

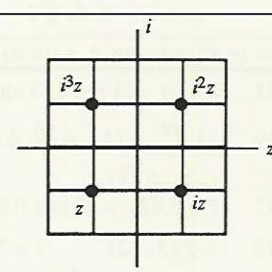
85. $I = \dfrac{V}{Z}$, so $V = IZ = (10 \text{ cis } 15°)(5 \text{ cis } 30°) = 50 \text{ cis } 45°$.

89. $P = 5 + 2i \approx 5.385 \text{ cis } 21.801°$
$Z = 1 - 4i \approx 4.123 \text{ cis } 284.036°$
$\sqrt{\dfrac{P}{Z}} \approx \sqrt{\dfrac{5.385 \text{ cis } 21.801°}{4.123 \text{ cis } 284.036°}} \approx (1.3061 \text{ cis}(-262.235°))^{1/2} \approx$
$\sqrt{1.3061}\text{cis}\left(\dfrac{-262.235°}{2}\right) \approx 1.143 \text{ cis}(-131.118°) \approx 0.75 + 0.86i$

93. (a) z: $-3 + i$
(b) iz: $i(-3 + i) = -3i + i^2 = -1 - 3i$
(c) i^2z: $(-1)(-3 + i) = 3 - i$
(d) i^3z: $(-i)(-3 + i) = 1 + 3i$



97. (a) z $\quad -1 - i$
(b) iz $\quad i(-1 - i) = -i - i^2 = 1 - i$
(c) i^2z $\quad (-1)(-1 - i) = 1 + i$
(d) i^3z $\quad (-i)(-1 - i) = -1 + i$



101. $r^{\frac{1}{n}} \operatorname{cis}\left(\frac{\theta}{n} + \frac{k \cdot 360°}{n}\right)$

$$= r^{\frac{1}{n}} \operatorname{cis}\left[\frac{\theta}{n} + \frac{(an+b)\cdot 360°}{n}\right] \qquad \text{Replace } k \text{ by } an+b.$$

$$= r^{\frac{1}{n}} \operatorname{cis}\left[\frac{\theta}{n} + \frac{an \cdot 360°}{n} + \frac{b \cdot 360°}{n}\right] = r^{\frac{1}{n}} \operatorname{cis}\left[\frac{\theta}{n} + a \cdot 360° + \frac{b \cdot 360°}{n}\right] = r^{\frac{1}{n}} \operatorname{cis}\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right]$$

$$= r^{\frac{1}{n}} \operatorname{cis}\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right]$$

$$= r^{\frac{1}{n}} \cos\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right] + i\, r^{\frac{1}{n}} \sin\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right]$$

$$\cos\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right] = \cos\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right)\cos(a \cdot 360°) - \sin\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right)\sin(a \cdot 360°)$$

$$= \cos\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right),$$

because $\cos(a{\cdot}360°) = 1$ and $\sin(a{\cdot}360°) = 0$, when a is an integer.

$$\sin\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right] = \sin\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right)\cos(a \cdot 360°) + \cos\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right)\sin(a \cdot 360°)$$

$$= \sin\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right),$$

because $\cos(a{\cdot}360°) = 1$ and $\sin(a{\cdot}360°) = 0$, when a is an integer.

Thus,

$$r^{\frac{1}{n}} \cos\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right] + i\, r^{\frac{1}{n}} \sin\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right]$$

$$= \quad r^{\frac{1}{n}} \cos\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + i\, r^{\frac{1}{n}} \sin\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right)$$

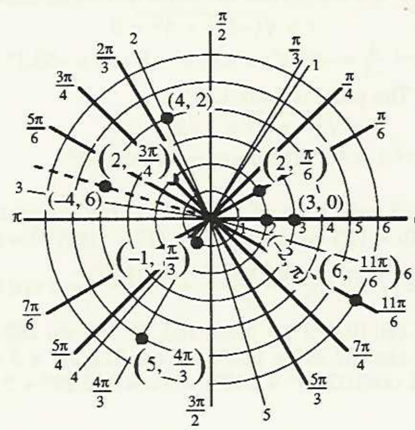
$$= \quad r^{\frac{1}{n}} \operatorname{cis}\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right), \text{ where } b < n.$$

This last expression is one of the previous roots.

Exercise 7–2

The points for problems 1 through 18 are plotted in the figures. Some special considerations are also listed.

9. $(-2, \pi) = (2, \pi - \pi) = (2, 0)$
11. $\left(-1, \frac{\pi}{3}\right) = \left(1, \frac{\pi}{3} + \pi\right) = \left(1, \frac{4\pi}{3}\right)$
13. $(4, 2)$; Note: $2 = 2{\cdot}\frac{180°}{\pi} \approx 115°$ (for plotting)
15. $(-5, 6) = (5, 6 - \pi)$; $6 - \pi = (6 - \pi)(\frac{180°}{\pi}) \approx 164°$. To plot $(-5, 6)$, plot a point 5 units from the center, at an angle about 164°.



Solutions to 1,3,5,7,9,11,13,15,17.

Many answers possible. To change the sign of r add an odd multiple of π to θ; for the rest add or subtract an even multiple of π.

21. $(6, \frac{11\pi}{6})$ $\quad (-6, \frac{11\pi}{6} - \pi)$, $\quad (6, \frac{11\pi}{6} + 2\pi)$ $\quad (6, \frac{11\pi}{6} - 2\pi)$
$(-6, \frac{5\pi}{6})$ $\quad (6, \frac{23\pi}{6})$ $\quad (6, -\frac{\pi}{6})$

Use $(r, \theta) = (r\cos\theta, r\sin\theta) = (x, y)$.

25. $(4, \frac{\pi}{2}) = (4\cos\frac{\pi}{2}, 4\sin\frac{\pi}{2}) = (4{\cdot}0, 4{\cdot}1) = (0, 4)$
29. $(4, \frac{4\pi}{3}) = (4\cos\frac{4\pi}{3}, 4\sin\frac{4\pi}{3}) = (4{\cdot}(-\frac{1}{2}), 4{\cdot}(-\frac{\sqrt{3}}{2})) =$
$(-2, -2\sqrt{3})$
33. $(3, 0.82) = (3\cos 0.82, 3\sin 0.82) \approx (2.05, 2.19)$
37. $(-2\sqrt{3}, -2)$ $\qquad r = \sqrt{(2\sqrt{3})^2 + 2^2} = \sqrt{4{\cdot}3 + 4} = 4.$
$\theta' = \tan^{-1}\left(\frac{-2}{-2\sqrt{3}}\right) = \tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{6}.$
Since $x = -2\sqrt{3} < 0$, and $\theta' > 0$, $\theta = \theta' - \pi = \frac{\pi}{6} - \pi = -\frac{5\pi}{6}.$
Thus the polar coordinates are $(4, -\frac{5\pi}{6})$.
41. $(-4, -4)$ $\qquad r = \sqrt{4^2 + 4^2} = 4\sqrt{2}$
$\theta' = \tan^{-1}(\frac{-4}{-4}) = \tan^{-1} 1 = \frac{\pi}{4}.$
Since $x < 0$, $\theta' > 0$, $\theta = \theta' - \pi = \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$
The point is $\left(4\sqrt{2}, -\frac{3\pi}{4}\right)$ in polar coordinates.
45. $(1, -4)$ $\qquad r = \sqrt{1^2 + 4^2} = \sqrt{17} \approx 4.12$
$\theta' = \tan^{-1}(-4) \approx -1.326$; $x > 0$ so $\theta = \theta'$.
$(4.12, -1.33)$

Use $x = r\cos\theta$, $y = r\sin\theta$.

49. $y = 4x$
$r\sin\theta = 4r\cos\theta$
$\sin\theta = 4\cos\theta$
$\dfrac{\sin\theta}{\cos\theta} = 4$
$\tan\theta = 4$

53. $y = mx + b,\ b \neq 0$
$r\sin\theta = mr\cos\theta + b$
$r\sin\theta - mr\cos\theta = b$
$r(\sin\theta - m\cos\theta) = b$
$r = \dfrac{b}{\sin\theta - m\cos\theta}$

57. $3x^2 + 2y^2 = 1$
$3(r\cos\theta)^2 + 2(r\sin\theta)^2 = 1$
$3r^2\cos^2\theta + 2r^2\sin^2\theta = 1$
$r^2(3\cos^2\theta + 2\sin^2\theta) = 1$
$r^2 = \dfrac{1}{3\cos^2\theta + 2\sin^2\theta}$
$r^2 = \dfrac{1}{3\cos^2\theta + 2(1 - \cos^2\theta)}$
$r^2 = \dfrac{1}{\cos^2\theta + 2}$

Use $\sin\theta = \dfrac{y}{r}$; $\cos\theta = \dfrac{x}{r}$; $r^2 = x^2 + y^2$.

61. $r = 2\sec\theta$
$r = \dfrac{2}{\cos\theta}$
$r = \dfrac{2}{\frac{x}{r}}$
$r = \dfrac{2r}{x}$
$rx = 2r$
$x = 2$
Assumes $r \neq 0$. It is not since $2\sec\theta$ is never 0.

65. $r^2 = \sin 2\theta$
$r^2 = 2\sin\theta\cos\theta$
$r^2 = 2\dfrac{y}{r}\cdot\dfrac{x}{r}$
$r^4 = 2xy$
$(x^2 + y^2)^2 = 2xy$
$x^4 + 2x^2y^2 + y^4 = 2xy$

69. $r = \dfrac{3}{1 - 2\sin\theta}$
$r(1 - 2\sin\theta) = 3$
$r(1 - \dfrac{2y}{r}) = 3$
$r - 2y = 3$
$r = 2y + 3$
$r^2 = (2y + 3)^2$
$x^2 + y^2 = 4y^2 + 12y + 9$

73. Consider a point $P = (r, \theta)$, where $r < 0$. Then $P = (-r, \theta + \pi)$, where $-r > 0$. Therefore, since $-r > 0$, $y = -r\sin(\theta + \pi)$ is true.

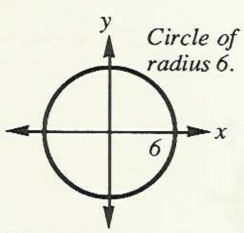
$y = -r\sin(\theta + \pi)$	A true statement.
$= -r(\sin\theta\cos\pi + \cos\theta\sin\pi)$	$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.
$= -r(\sin\theta\,(-1) + \cos\theta\,(0)) = -r(-\sin\theta) = r\sin\theta$.	

Thus $y = r\sin\theta$, even if $r < 0$.

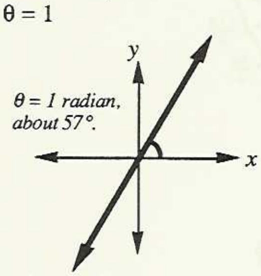
77. $r = 1 + 2\sin 2\theta$
$r = 1 + 2(2\sin\theta\cos\theta)$
$r = 1 + 4\dfrac{y}{r}\cdot\dfrac{x}{r}$
$r^3 = r^2 + 4xy$
$r^6 = (r^2 + 4xy)^2$
$(r^2)^3 = (x^2 + y^2 + 4xy)^2$
$(x^2 + y^2)^3 = (x^2 + y^2 + 4xy)^2$

Exercise 7–3

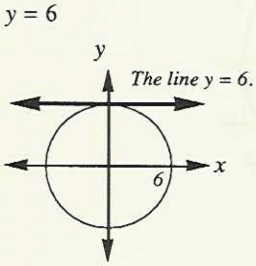
1. $r = 6$



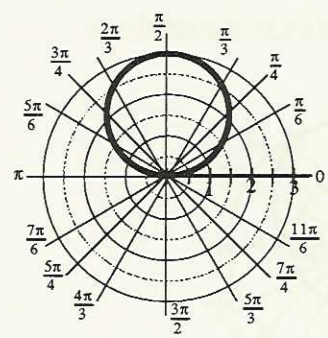
5. $\theta = 1$



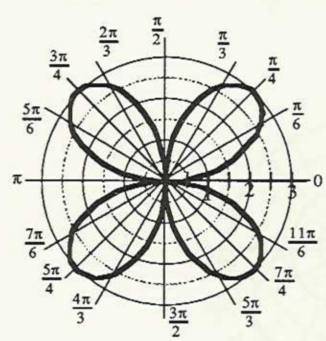
9. $r\sin\theta = 6$
$y = 6$



13. $r = 3\sin\theta$



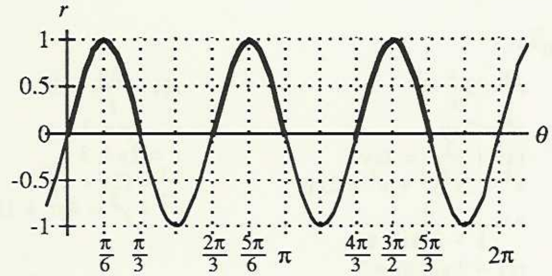
17. $r = 3\sin 2\theta$

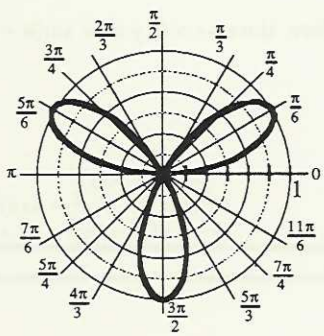


20. $r = \sin 3\theta$

The rectangular coordinate graph of $r = \sin 3\theta$ is shown below. The negative lobes can be ignored because shifting them by π units and flipping them about the θ axis causes them to overlap the positive lobes.

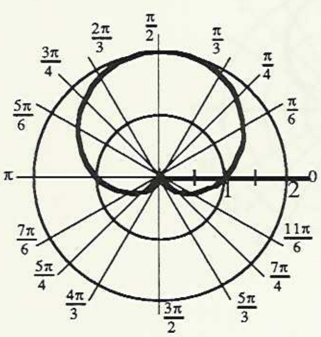
We see there are lobes between 0 and $\frac{\pi}{3}$, with maximum at $\frac{\pi}{6}$, $\frac{2\pi}{3}$ and π, with maximum at $\frac{5\pi}{6}$, and $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$, with maximum at $\frac{3\pi}{2}$.



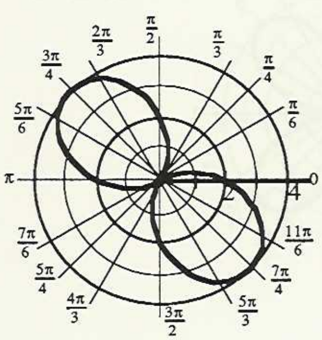


These lobes produce the polar graph shown.

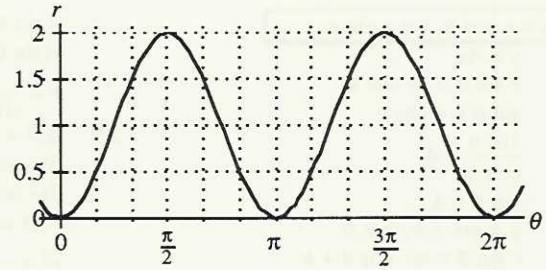
21. $r = 1 + \sin\theta$

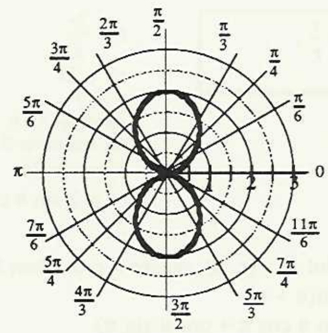


25. $r = 2 - \sin 2\theta$



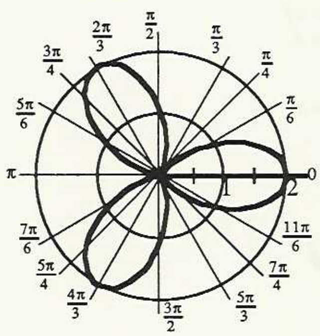
26. $r = 1 - \cos 2\theta$



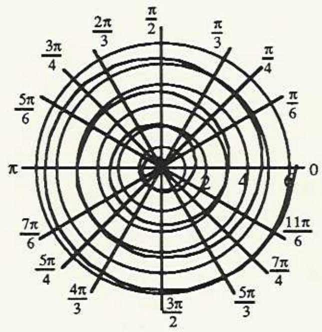


The rectangular graph shows two lobes, one between 0 and π, with maximum at $\frac{\pi}{2}$, and one between π and 2π, with maximum at $\frac{3\pi}{2}$. The two positive lobes produce the polar graph shown. The two lobes are not circles, which would require algebra beyond the scope of this text to show. Of the problems in this text, only those equations categorized in the text as producing circles will in fact produce circles.

27. $r = 1 + \cos 3\theta$



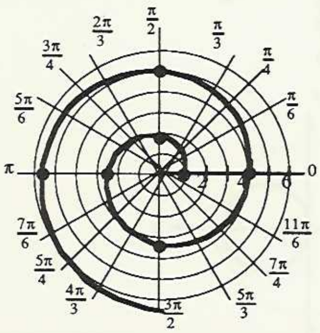
31. $r = \frac{\theta}{4}, \theta > 0$



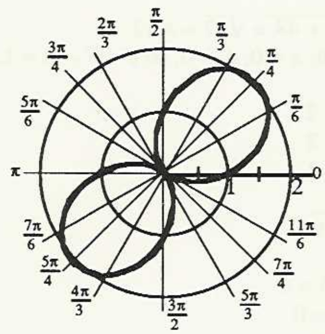
32. $r = 1 + \frac{\theta}{2}$

This graph is not periodic, and thus an analysis of its rectangular graph is less helpful than in the case where the trigonometric functions are involved. We simply plot points to obtain the graph. It is clear that as the value of θ increases, r increases. This property produces a spiral, which starts at $r = 1$.
A table of values and the graph are shown.

θ	r
0	1
$\frac{\pi}{2}$	1.8
π	2.6
$\frac{3\pi}{2}$	3.4
2π	4.1
$\frac{5\pi}{2}$	4.9
3π	5.7

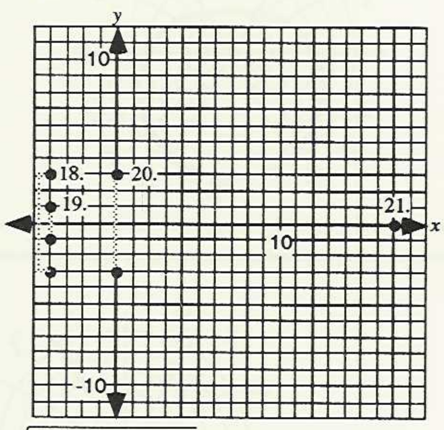


35. $r = 1 + \sin 2\theta$



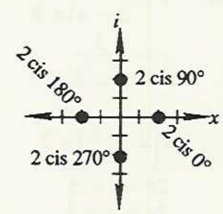
Chapter 7 Review

1. real: 3; im: –1
3. real: 12; im: 0
5. $2 - 4i - 3 + 2i = -1 - 2i$
7. $15 + 60i - 12i - 48i^2$
   $15 + 48i + 48$
   $63 + 48i$
9. $8i - 12i^2$
   $8i + 12$
   $12 + 8i$
11. $\frac{10 - 4i}{5 + 2i} \cdot \frac{5 - 2i}{5 - 2i} = \frac{42 - 40i}{29} = \frac{42}{29} - \frac{40}{29}i$
13. $\frac{8 + 6i}{14i} \cdot \frac{-i}{-i} = \frac{-8i - 6i^2}{-14i^2} = \frac{6 - 8i}{14} = \frac{3}{7} - \frac{4}{7}i$
15. $(2 + i)(3 - (3 - 2i)) = (2 + i)(2i) = 4i + 2i^2 = -2 + 4i$
17. $\frac{(2 + i)((2 + i) - (3 - 2i))}{2(3 - 2i)} = \frac{(2 + i)(-1 + 3i)}{(6 - 4i)}$
    $= \frac{-50 + 10i}{52} = -\frac{25}{26} + \frac{5}{26}i$
18. $-4 - 3i$
19. $-4 - i$
20. $-3i$
21. 17

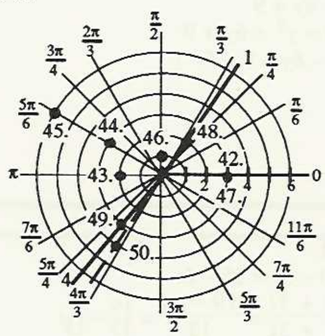


25. $\sqrt{3} - i$ $\quad r = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$
    $\theta' = \tan^{-1}(\frac{y}{x}) = \tan^{-1}(-\frac{1}{\sqrt{3}}) = -30°$; $a > 0$ so $\theta = \theta'$
    2 cis(–30°)
27. $-4i$ $\quad r = \sqrt{0^2 + (-4)^2} = 4$
    $\theta = -90°$ (plot the point $0 - 4i$ to see this)
    4 cis(–90°)
29. 3 cis 243° = 3 cos 243° + 3 sin 243° ≈ $\quad -1.4 - 2.7i$
31. $10 \text{ cis } 330° = 10 \cos 330° + 10i \sin 330° = 10 \cdot \frac{\sqrt{3}}{2} + 10i\left(-\frac{1}{2}\right)$
    $= 5\sqrt{3} - 5i$
33. (2 cis 18°)(6.5 cis 122°) = 2•6.5 cis(18° + 122°) = 13 cis 140°
35. $\frac{50 \text{ cis } 45°}{100 \text{ cis } 9°} = \frac{50}{100} \text{cis}(45° - 9°) = \frac{1}{2} \text{cis } 36°$
37. $(2 \text{ cis } 150°)^4 = 2^4 \text{ cis}(4 \cdot 150°) = 16 \text{ cis } 600° =$
    16 cis(600° – 720°) = 16 cis(–120°)
39. 16 = 16 cis 0°; evaluate
    $16^{1/4} \text{ cis}\left(\frac{0°}{4} + \frac{k \cdot 360°}{4}\right) =$
    2cis(k•90°) for k = 0, 1, 2, 3.

$k = 0$:	2 cis 0° =	2
$k = 1$:	2 cis 90° =	$2i$
$k = 2$:	2 cis 180° =	-2
$k = 3$:	2 cis 270° =	$-2i$



41. $I = \frac{130 \text{ cis } 25°}{30 \text{ cis } 75°} = \frac{130}{30} \text{cis}(25° - 75°) = 4\frac{1}{3} \text{ cis}(-50°)$



Answers to 42 through 50.

53. $(-2, \frac{5\pi}{3}) = (-2 \cos \frac{5\pi}{3}, -2 \sin \frac{5\pi}{3}) = (-2 \cdot \frac{1}{2}, -2(-\frac{\sqrt{3}}{2})) = (-1, \sqrt{3})$
55. (5, 2) = (5 cos 2, 5 sin 2) ≈ (–2.08, 4.55)
57. (2, 1) $\quad r = \sqrt{2^2 + 1^2} = \sqrt{5} \approx 2.24$
    $\theta = \theta' = \tan^{-1} \frac{1}{2} \approx 0.46$; (2.24, 0.46)

59. $(-1, -4)$ $\quad r = \sqrt{1^2 + 4^2} = \sqrt{17} \approx 4.12$
$\theta' = \tan^{-1} 4 \approx 1.326$; $x < 0$, $\theta' > 0$, so $\theta = \theta' - \pi \approx 1.326 - \pi \approx -1.82$; $(4.12, -1.82)$

61. $y = 4x + 2$
$r \sin\theta = 4r\cos\theta + 2$
$r\sin\theta - 4r\cos\theta = 2$
$r(\sin\theta - 4\cos\theta) = 2$
$r = \dfrac{2}{\sin\theta - 4\cos\theta}$

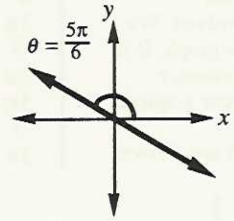
63. $y^2 - 3x = 0$
$(r\sin\theta)^2 - 3r\cos\theta = 0$
$r^2\sin^2\theta - 3r\cos\theta = 0$
$r\sin^2\theta - 3\cos\theta = 0$
$r\sin^2\theta = 3\cos\theta$
$r = \dfrac{3\cos\theta}{\sin^2\theta}$
$r = 3\dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{1}{\sin\theta}$
$r = 3\cot\theta\csc\theta$ $\quad$ Alternate form of answer.

65. $r = 2\sec\theta$
$r = 2\cdot\dfrac{1}{\cos\theta}$
$r = 2\cdot\dfrac{r}{x}$
$1 = \dfrac{2}{x}$ $\quad$ Divide each member by r.
$x = 2$

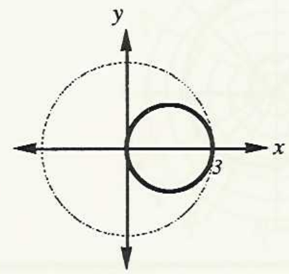
67. $r^2 = \tan\theta$
$x^2 + y^2 = \dfrac{y}{x}$
$x^3 + xy^2 = y$

69. $r = \dfrac{3}{2 - \sin\theta}$
$r = \dfrac{3}{2 - \frac{y}{r}}$
$r\left(2 - \dfrac{y}{r}\right) = 3$
$2r - y = 3$
$2r = y + 3$
$(2r)^2 = (y + 3)^2$
$4r^2 = y^2 + 6y + 9$
$4(x^2 + y^2) = y^2 + 6y + 9$
$4x^2 + 3y^2 - 6y - 9 = 0$

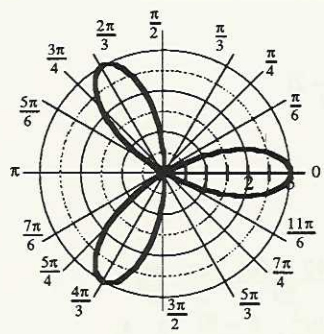
71.



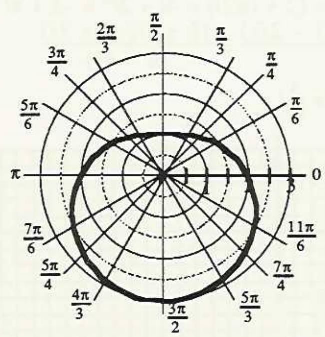
73. $r = 3\cos\theta$



75. $r = 3\cos 3\theta$



77. $r = 2 - \sin\theta$



Chapter 7 Test

1. $12 - 4i + 3 - 2i = 15 - 6i$

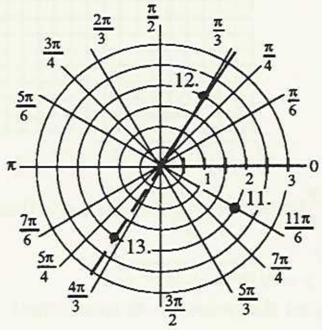
3. $\dfrac{2 - 5i}{2 - 3i}\cdot\dfrac{2 + 3i}{2 + 3i} = \dfrac{19 - 4i}{13} = \dfrac{19}{13} - \dfrac{4}{13}i$

5. $4 - 5i$ $\quad r = \sqrt{4^2 + 5^2} = \sqrt{41} \approx 6.40$
$\theta = \theta' - \tan^{-1}(-\frac{5}{4}) \approx -51.3$ $\quad$ $\theta = \theta'$ because $a > 0$
$6.40 \text{ cis}(-51.3°)$

7. $(2 \text{ cis } 325°)(7 \text{ cis } 145°) = 2{\cdot}7 \text{ cis } (325° + 145°) = 14 \text{ cis } 470°$
$= 14 \text{ cis}(470° - 360°) = 14 \text{ cis } 110°.$

9. $(3 \text{ cis } 150°)^3 = 3^3 \text{ cis}(3{\cdot}150°) = 27 \text{ cis } 450°$
$= 27 \text{ cis}(450° - 360°) = 27 \text{ cis } 90°.$

11. $(2, \frac{11\pi}{6})$ $\qquad$ 12. $(2, \frac{\pi}{3})$ $\qquad$ 13. $(-2, 1)$



15. $(-5, 2) = (-5\cos 2, -5\sin 2) \approx (2.08, -4.55)$

17. $(-5,5)$

$r = \sqrt{25+25} = 5\sqrt{2}$

$\theta' = \tan^{-1}\frac{5}{-5} = \tan^{-1}(-1) = -\frac{\pi}{4}$

Since $x < 0$, $\theta' < 0$ then θ

$= \theta' + \pi = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$

19. $2y^2 - x = 5$

$2(r \sin \theta)^2 - r \cos \theta = 5$

$2r^2 \sin^2\theta - r \cos \theta = 5$

21. $r^2 = \cos 2\theta$

$x^2 + y^2 = \cos^2\theta - \sin^2\theta$

$x^2 + y^2 = \frac{x^2}{r^2} - \frac{y^2}{r^2}$

$x^2 + y^2 = \frac{x^2 + y^2}{r^2}$

$x^2 + y^2 = \frac{x^2 - y^2}{x^2 + y^2}$

$(x^2 + y^2)^2 = x^2 - y^2$

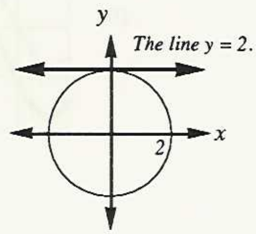
$x^4 + 2x^2y^2 + y^4 = x^2 + y^2$

23. $r = 2 \csc \theta$

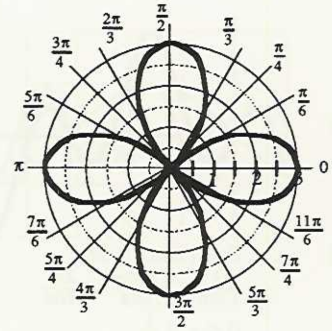
$r = \frac{2}{\sin \theta}$

$r \sin \theta = 2$

$y = 2$

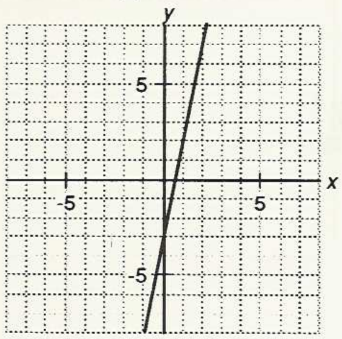


24. $r = 3 \cos 2\theta$

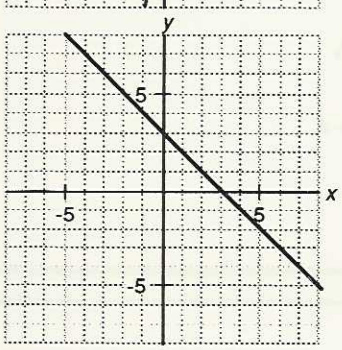


Exercises for Appendix A

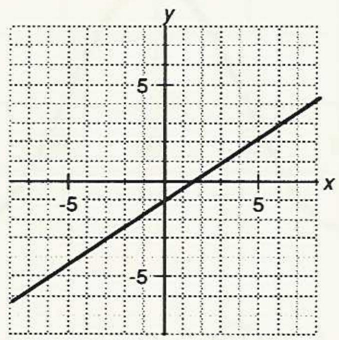
1.



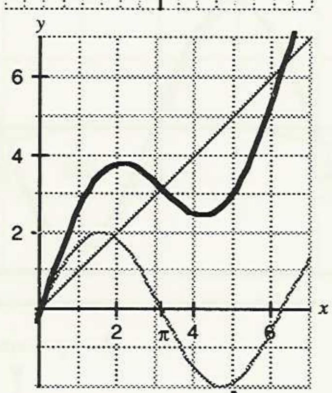
3.



5.



7.



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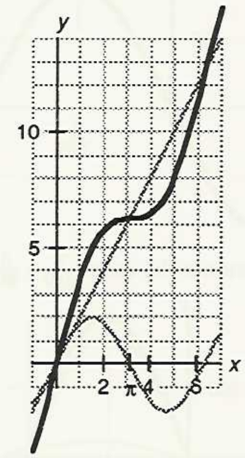

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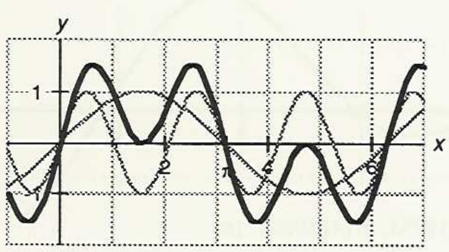

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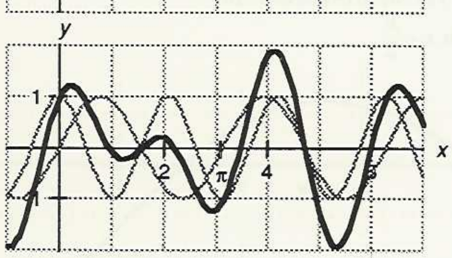

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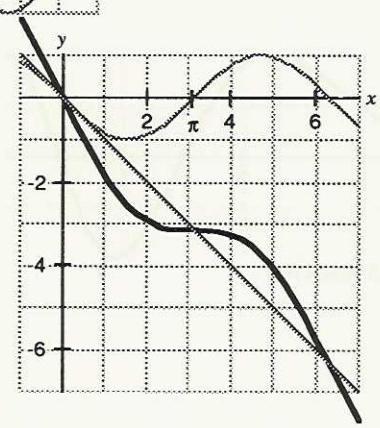
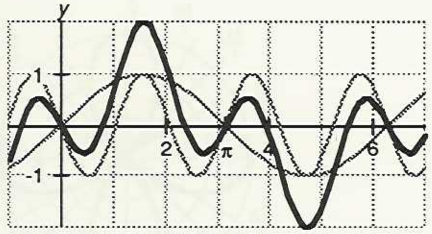
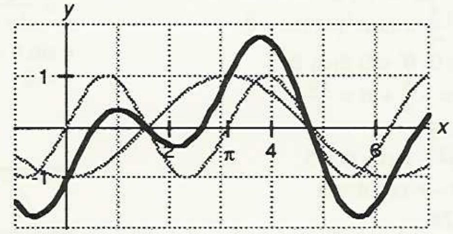


17. 

19. 

21. a. $0 \le 120\pi x \le 2\pi$

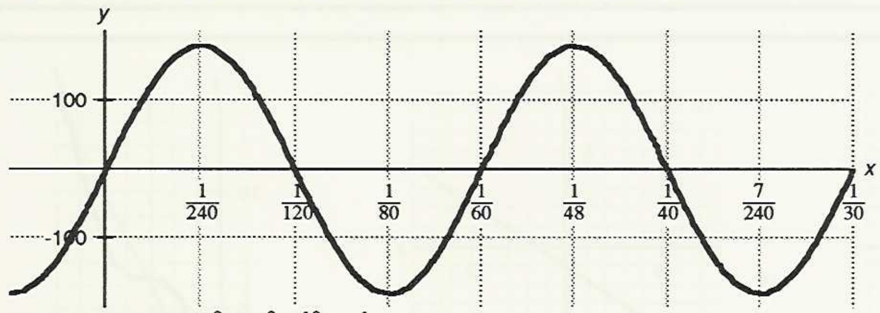
$\frac{0}{120\pi} \le \frac{120\pi x}{120\pi} \le \frac{2\pi}{120\pi}$

$0 \le x \le \frac{1}{60}$

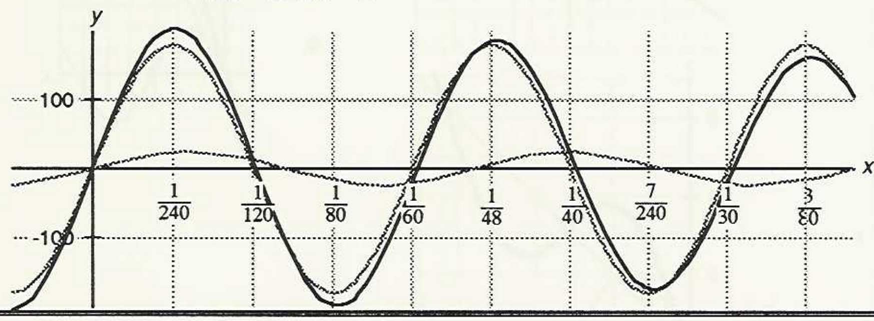
Two cycles are $2 \cdot \frac{1}{60} = \frac{1}{30}$ (seconds)

$\frac{1}{30} \div \frac{1}{240} = \frac{1}{30} \cdot \frac{240}{1} = 8$, so there are 8 tick marks for the two cycles. These are multiples of $\frac{1}{240}$:

$0, \frac{1}{240}, \frac{2}{240} = \frac{1}{120}, \frac{3}{240} = \frac{1}{80}, \frac{4}{240} = \frac{1}{60}, \frac{5}{240} = \frac{1}{48}, \frac{6}{240} = \frac{1}{40}, \frac{7}{240}, \frac{8}{240} = \frac{1}{30}$

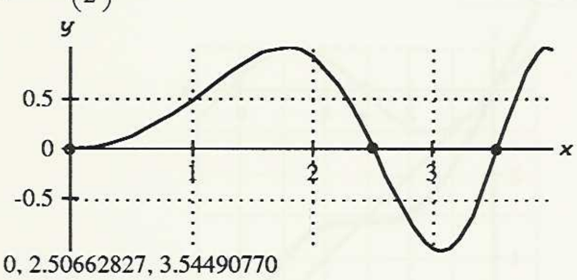


b. Continue the units: $\frac{9}{240} = \frac{3}{80}, \frac{10}{240} = \frac{1}{24}$



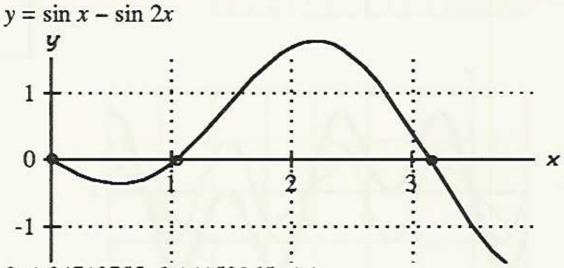
Exercises for Appendix B

1. $y = \sin\left(\frac{x^2}{2}\right)$



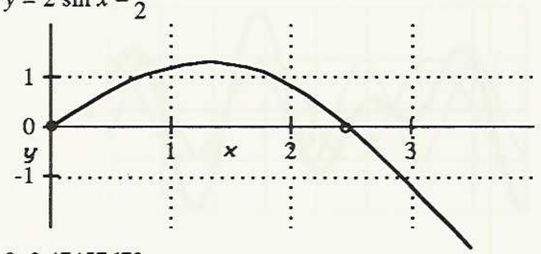
0, 2.50662827, 3.54490770

3. $y = \sin x - \sin 2x$



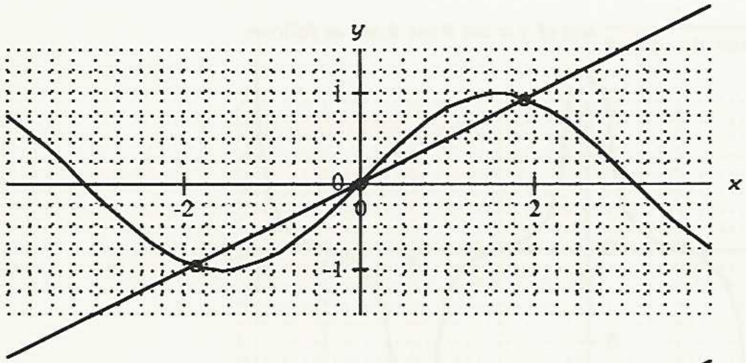
0, 1.04719755, 3.14159265 (π)

5. $y = 2 \sin x - \frac{x}{2}$



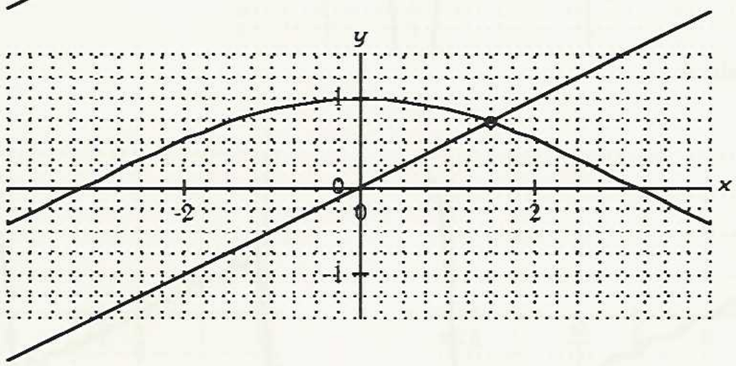
0, 2.47457679

7. $y = \sin x$; $y = \frac{x}{2}$



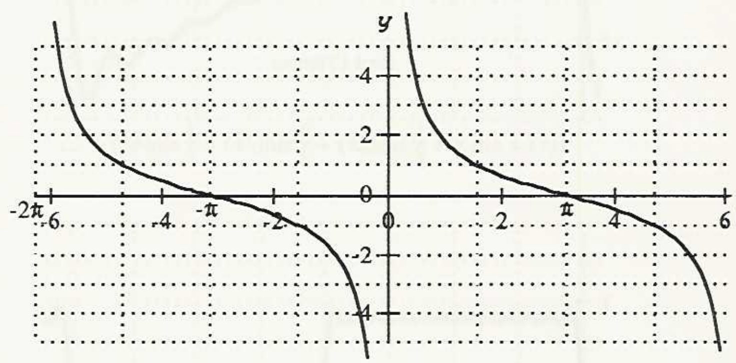
0, ± 1.89549427

9. $y = \cos \frac{x}{2}$; $y = \frac{x}{2}$

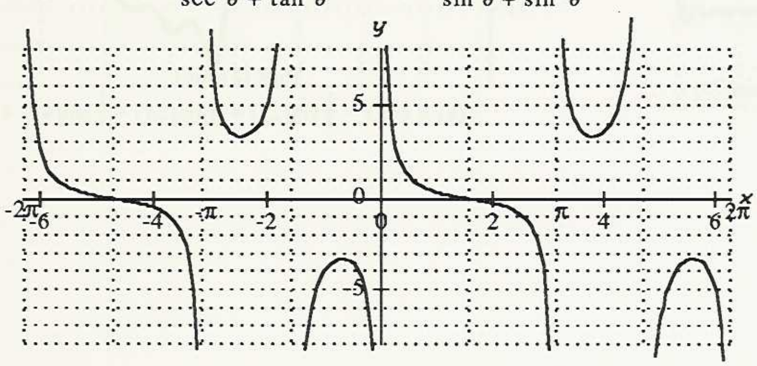


1.47817027

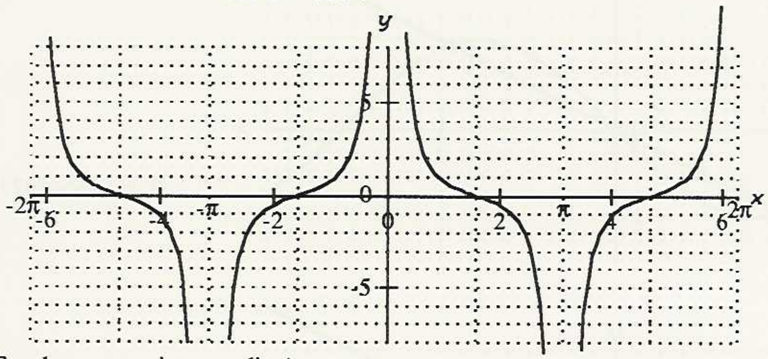
11. The graph of $y = \csc \theta + \cot \theta$, and $y = \dfrac{1 + \cos \theta}{\sin \theta}$ both look like:



13. The graphs of $y = \dfrac{\csc \theta}{\sec \theta + \tan \theta}$ and of $y = \dfrac{\cos \theta}{\sin \theta + \sin^2 \theta}$ both look like the following.

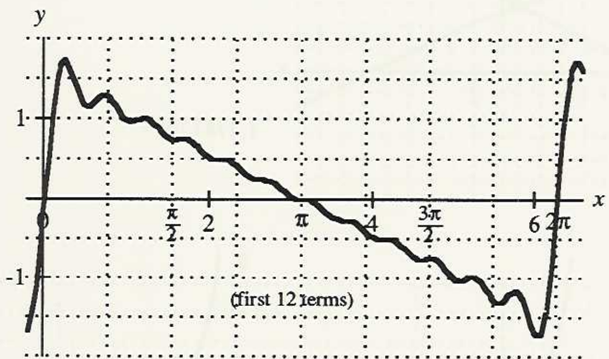


15. The graph of both $y = \dfrac{1}{\sec \theta - \cos \theta}$ and of $y = \cot \theta \csc \theta$ are as follows.

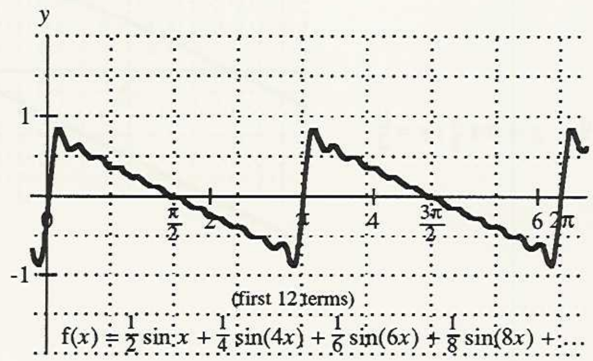


17, 19 See the answers in appendix A.

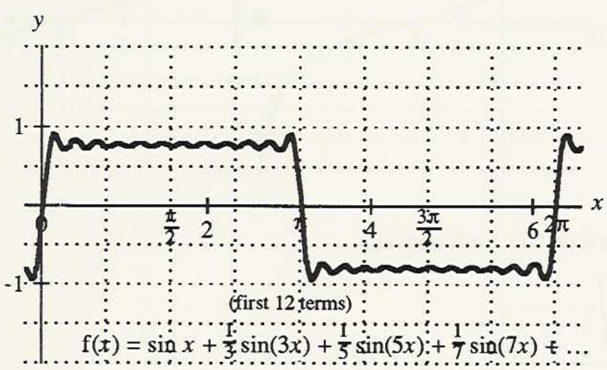
21.



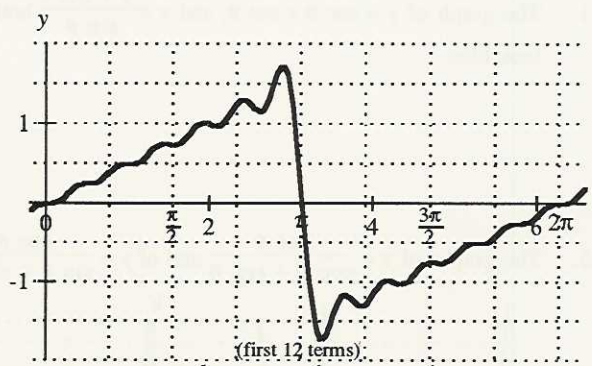
$f(x) = \sin x + \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) + \frac{1}{4}\sin(4x) + \ldots$



$f(x) = \frac{1}{2}\sin x + \frac{1}{4}\sin(4x) + \frac{1}{6}\sin(6x) + \frac{1}{8}\sin(8x) + \ldots$



$f(x) = \sin x + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x) + \ldots$



$f(x) = \sin x - \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) - \frac{1}{4}\sin(4x) + \ldots$

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